Data-Driven Modelling and Control

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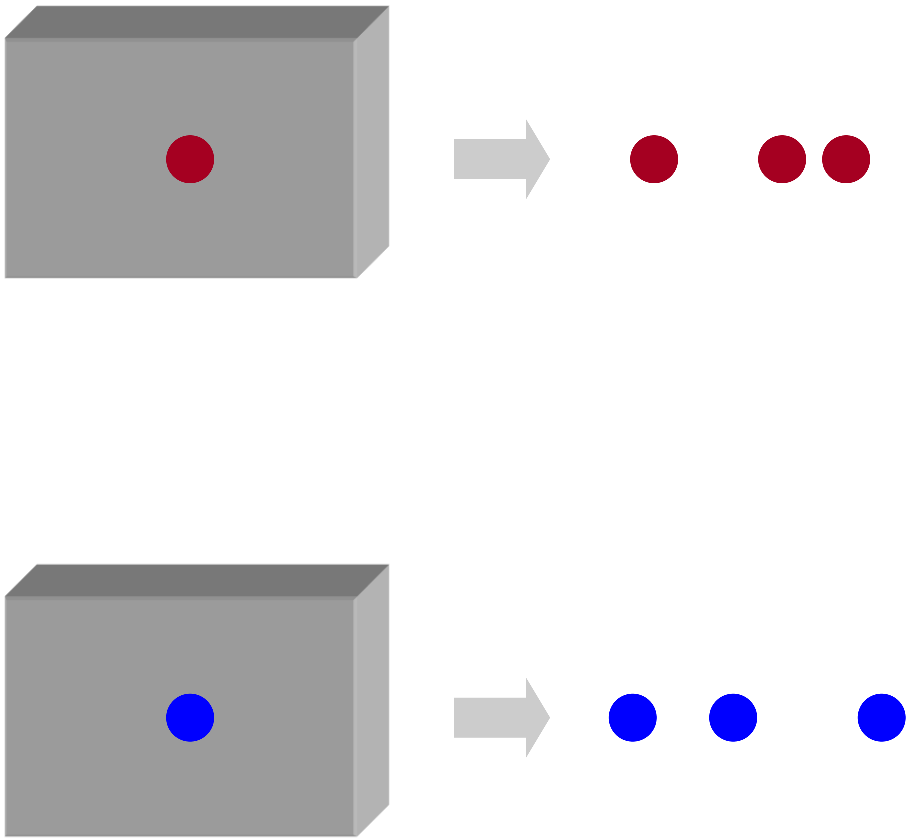

Modeling, Analysis and Control of **Output Dynamics** of Production Systems

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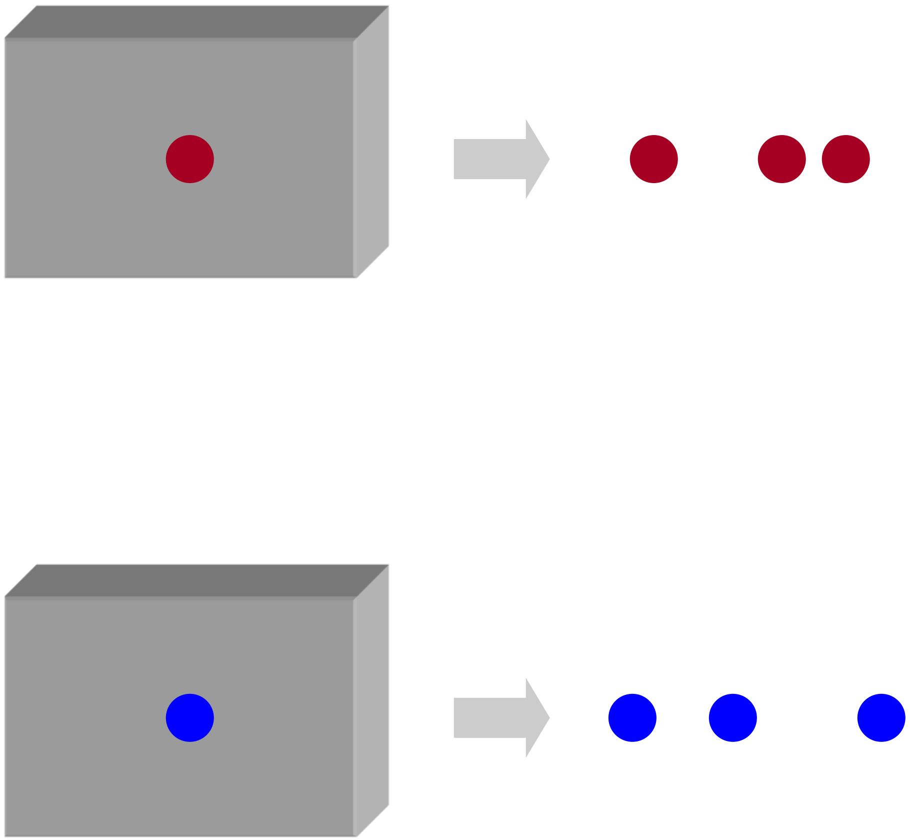
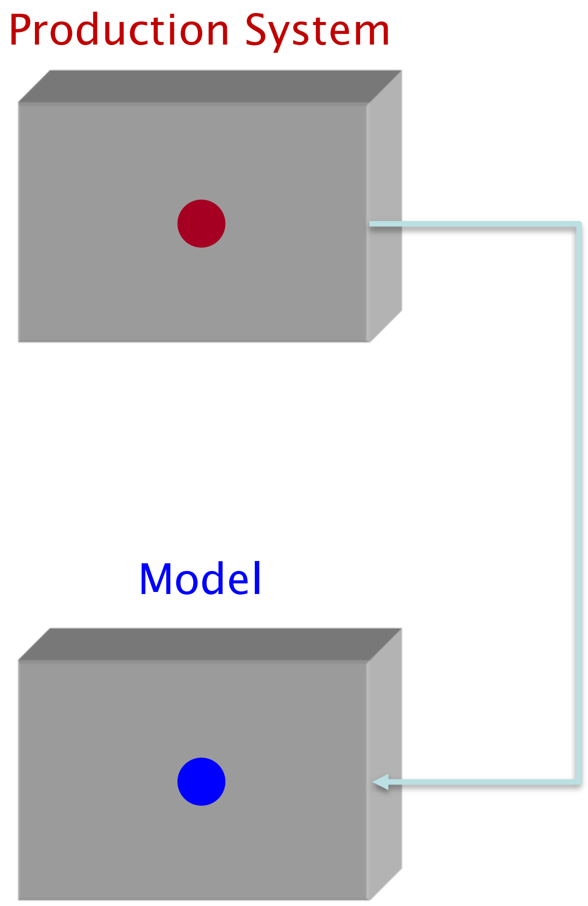
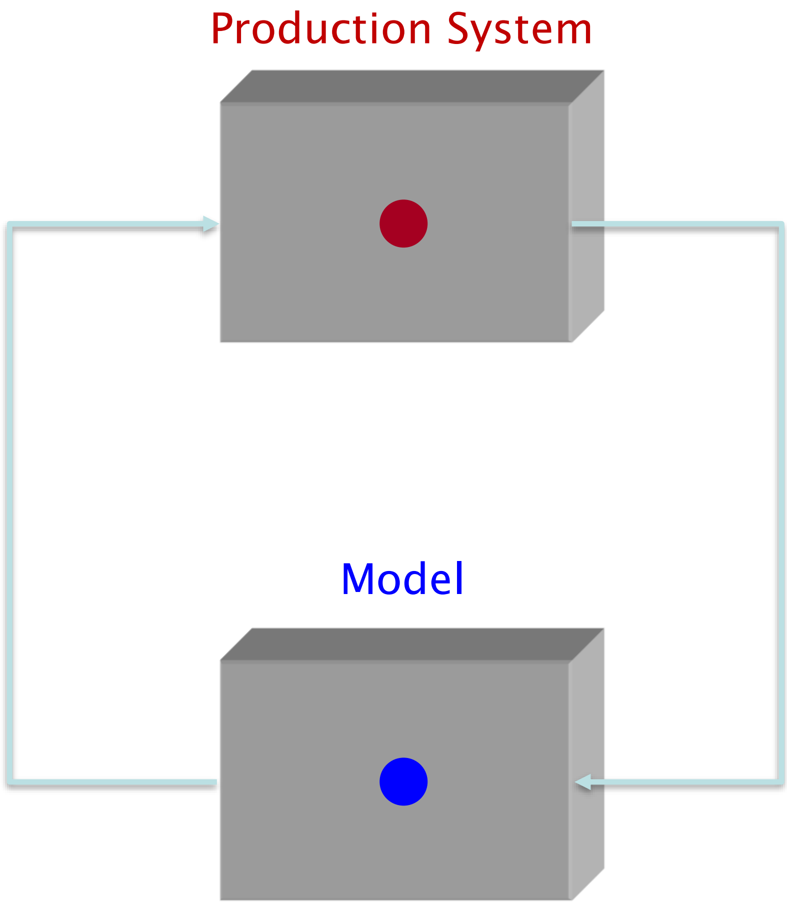


Research Questions
Modelling and Analysis



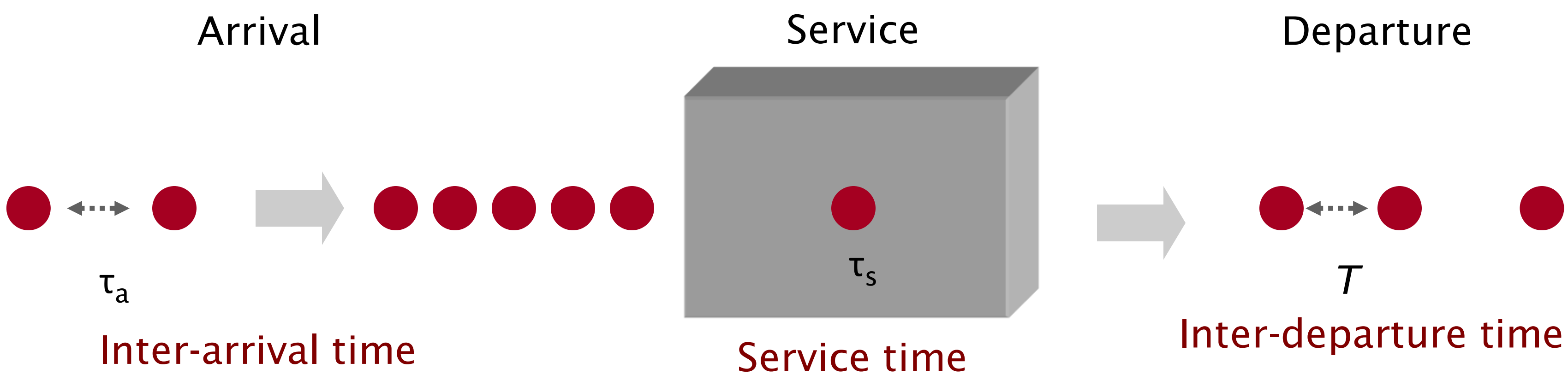
- Capturing **output dynamics** from production systems accurately in analytical models
- **Analytical methods** to analyze the statistical properties of output dynamics from production systems
- Understanding **the effects** of system parameters on the output dynamics
- Understanding **the impact** of autocorrelation on the performance of production systems

Research Questions
Data-Driven Modelling and Control



- **Fitting** models based on observed **data**
- **Controlling** production systems effectively by using models that capture output dynamics
- Controlling production systems by **modulating** the input arrival streams

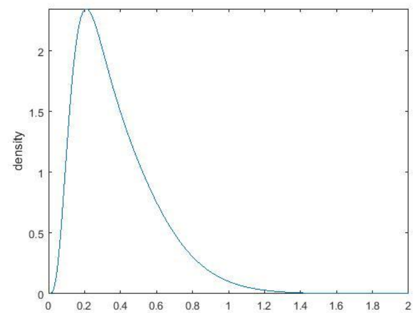
Output Dynamics can be represented by mean, variance, distribution, and auto-correlation

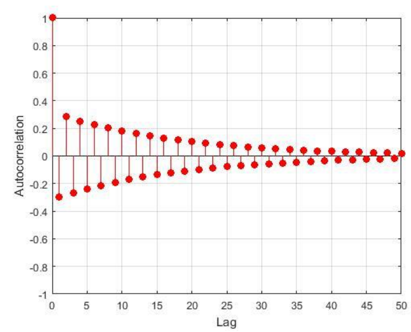


$E[N(t)]$ $Var[N(t)]$

E V

$P[N(t) \text{£} n - 1]$





$E[T_n]$ $Var[T_n]$

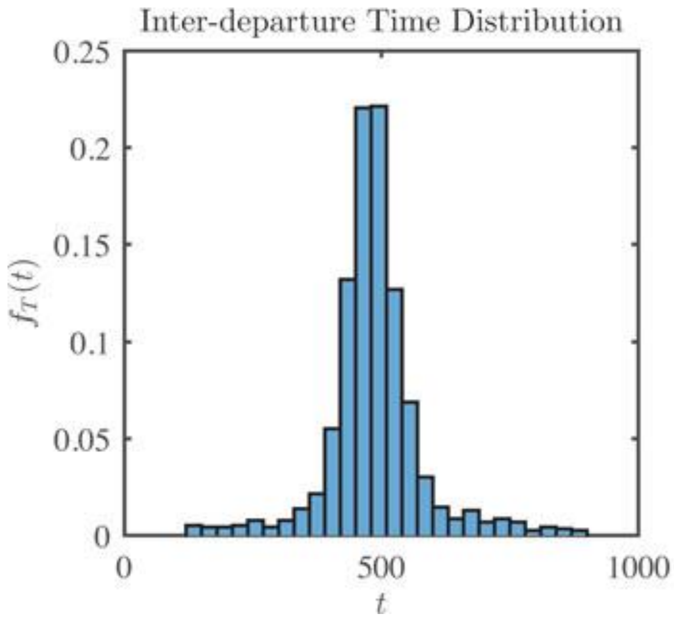
$E(T)$ $Var(T)$

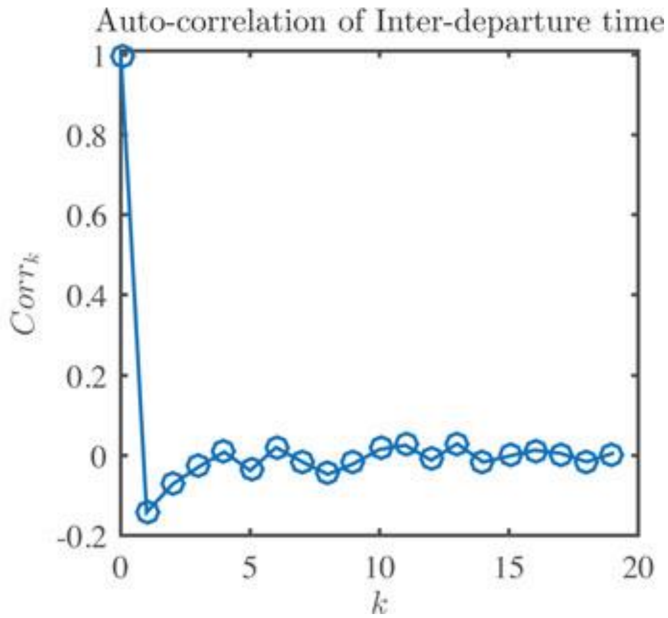
$F_T(t)$

$$Corr(T)_k = \frac{Cov(T_i, T_{i+k})}{Var(T)}$$

Output Dynamics
Automotive Assembly Line

Frequency distribution of the inter-departure time and sample autocorrelation of inter-departure times for a car assembly line





(Tan and Lagershausen 2017)

Output Dynamics
Semiconductor Manufacturing

Sample autocorrelation of cycle times for two fabs

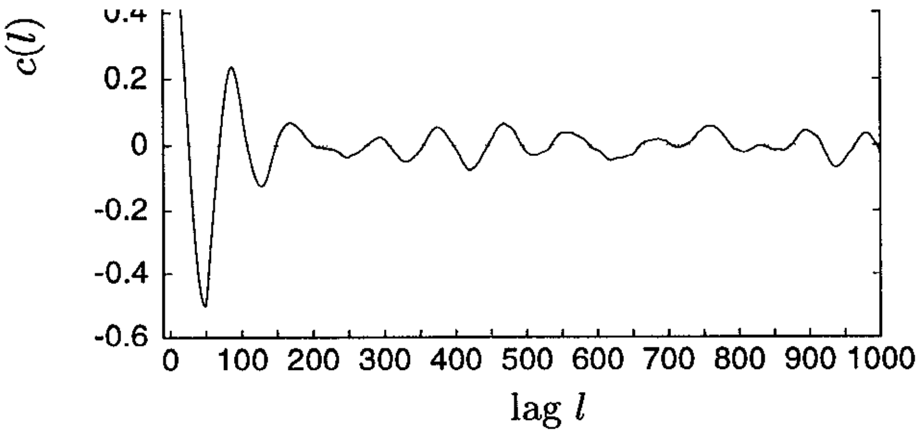


Figure 3: Autocorrelation of Cylce Times in Wein's Semiconductor Fabrication Facility

We conclude that due to the application of the closed

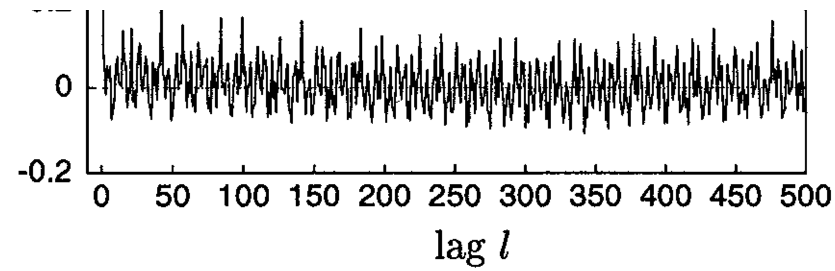


Figure 6: Autocorrelation of Cycle Times, MIMAC Data Set 3

Like for Wein's fabrication facility we find an oscillating autocorrelation function, here within an interval of [−0.1; 0.2] and with a much higher frequency. Again we use periodograms to obtain more information about periodic effects in the simulation output.

(Schömig and Mittler, 1995)

Production System vs Model

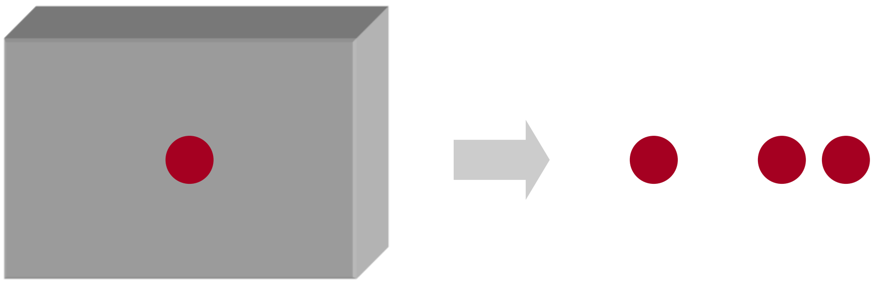
“*Mathematical Twin*”

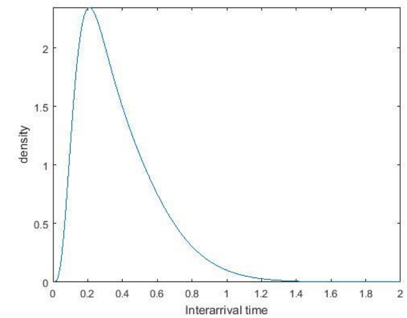
$E(T)$ $Var(T)$

$F_T(t)$

Production System

Departure

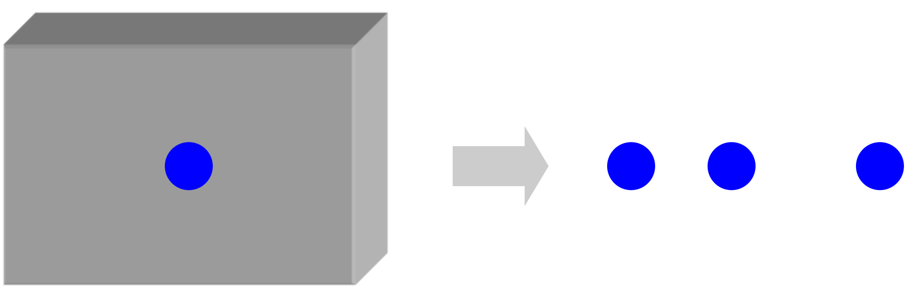


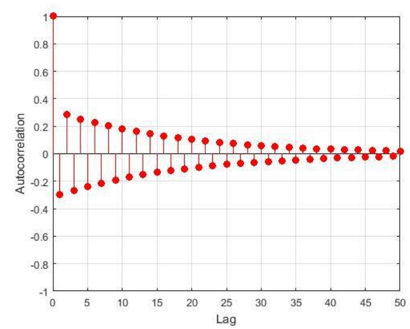


$$Corr(T)_k = \frac{Cov(T_i, T_{i+k})}{Var(T)}$$

Model

Departure





Output Dynamics
can be captured with Markov Arrival Processes (MAPs)

$\mathrm{MAP}(D_0, D_1)$

$$Q = \begin{bmatrix} D_0 & D_1 & & & \\ & D_0 & D_1 & & \\ & & D_0 & D_1 & \\ & & & \ddots & \ddots \\ & & & & \end{bmatrix}$$

Analysis of MAPs

Lakatos et al. (2013)
He (2014)
Buchholz et al. (2014)

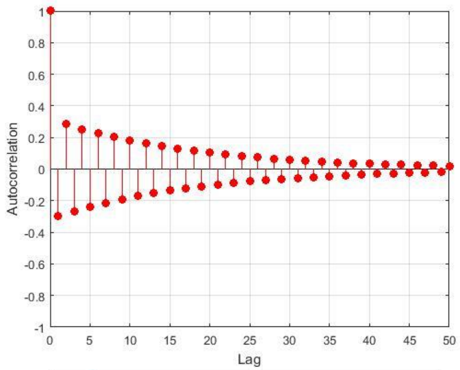
Methods to fit
phase-type distributions
moments and
autocorrelations

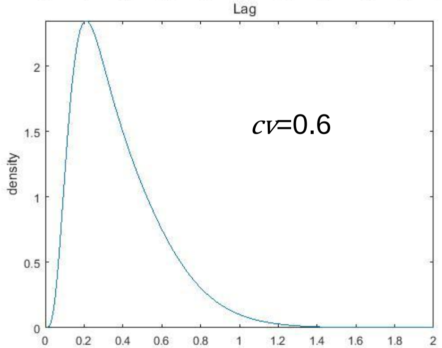
Bodrag et. al. (2008)
Horváth (2013)
Okamura and Dohi (2016)

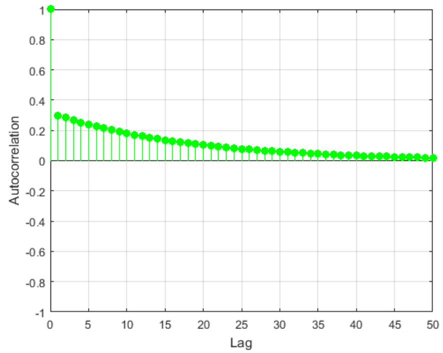
Inter-Departure Time Dynamics
Same Distribution: Negative, 0, Positive autocorrelation

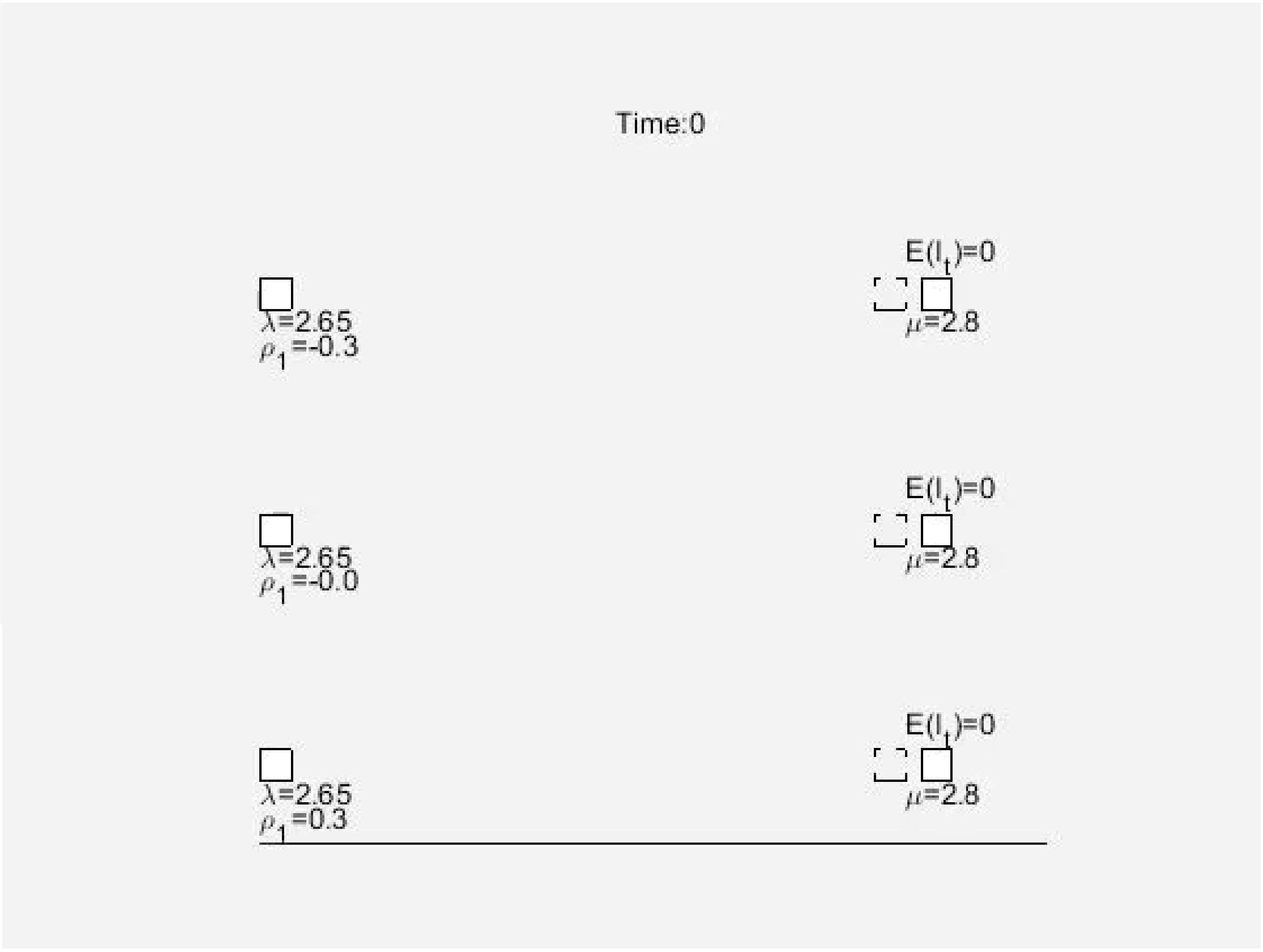
Arrival: MAP

Service: Erlang (4)

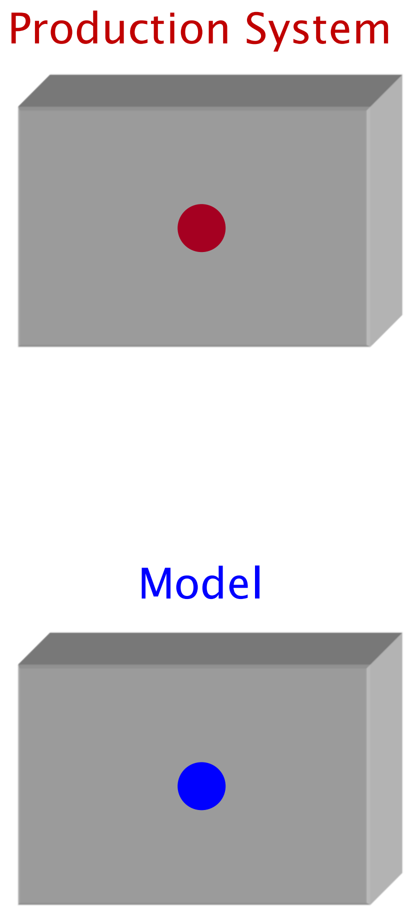








Research Questions
Modelling and Analysis



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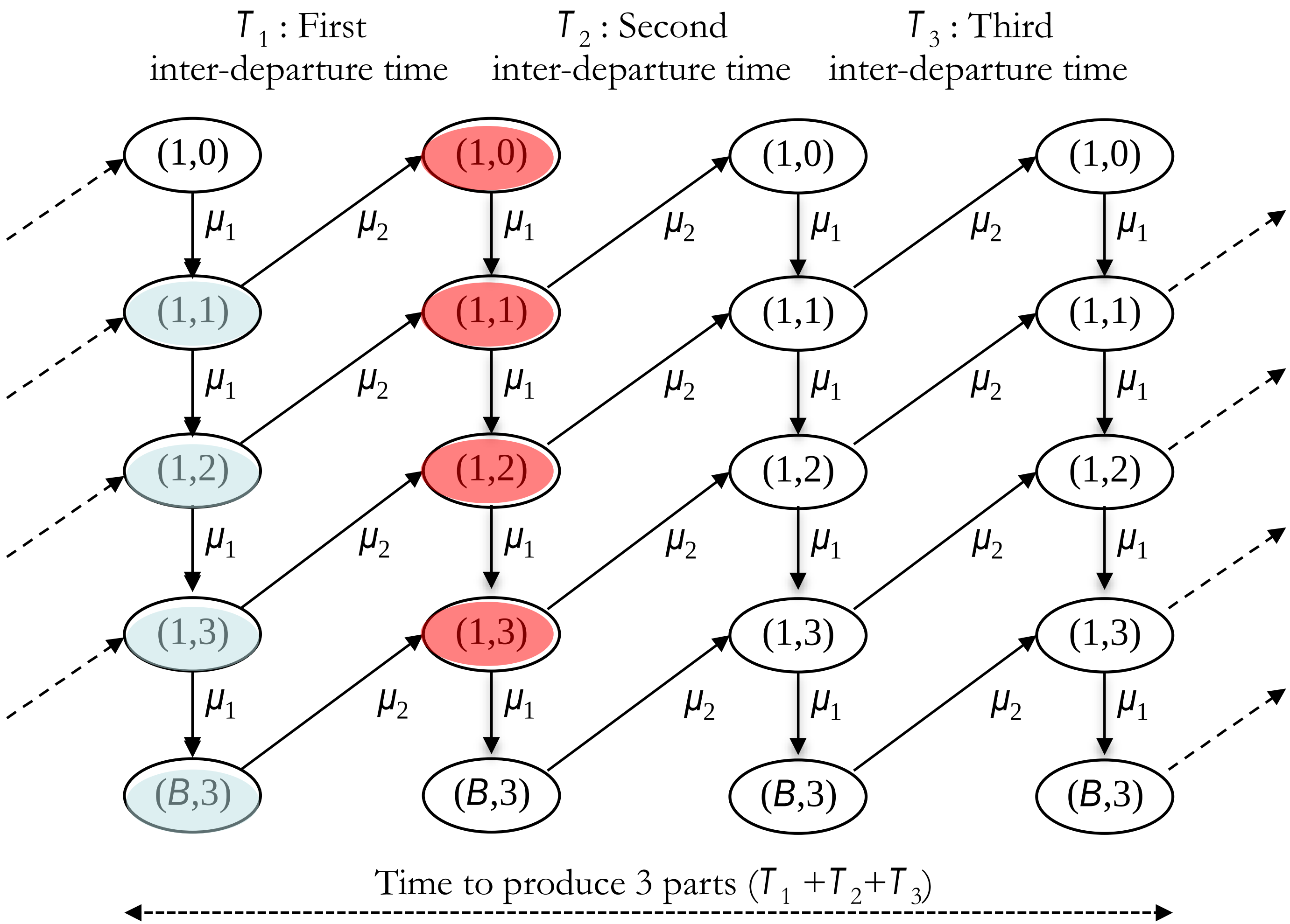
The inter-departure time distribution
can be determined analytically



The inter-departure time distribution
can be determined analytically

Start with a **CTMC representation** of a Production System

$$X(t) \in S \qquad \{X(t), t \geq 0\}$$

$$Q = \{q_{i,j}\}$$

Identify events that lead to a **departure**

$$G_d \qquad R_d = Q \circ G_d$$

$$Q_d = Q - R_d$$

First-passage time analysis yields the distribution, mean, and variance of the inter-departure time

$$F_T(t) = 1 - \pi^{\text{entry}} e^{Q_d t} u$$

$$E(T) = -\pi^{\text{entry}} Q_d^{-1} u,$$

$$Var(T) = 2\pi^{\text{entry}} Q_d^{-2} u - \left(\pi_d^{\text{entry}} Q_d^{-1} u\right)^2$$

$$\pi^{\text{entry}} \left(I + Q_d^{-1} R_d\right) = 0$$

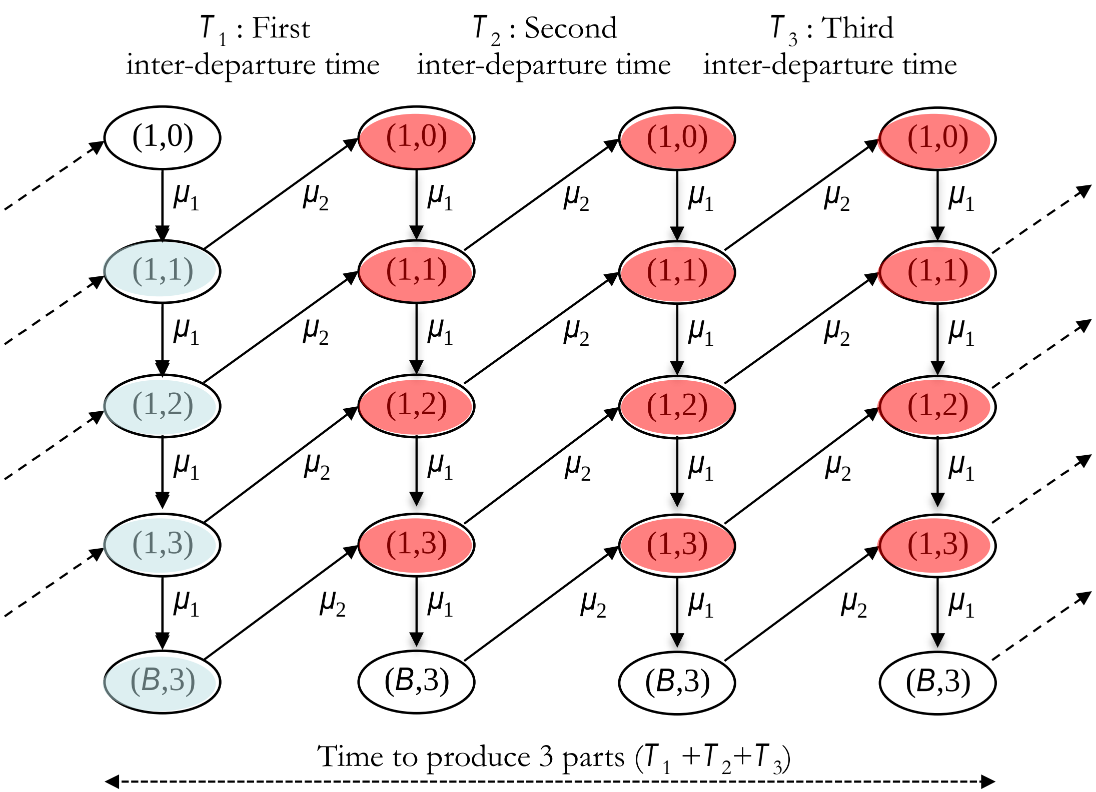
$$\pi^{\text{entry}} u = 1$$

(Lagershausen and Tan 2015)

Departure Autocorrelation

for Production Systems can be determined analytically by

Extending this approach to analyze **sequence of *k* inter-departure times** and determining **the variance of the time to produce *k* products**



Departure Autocorrelation
for Production Systems can be determined analytically

Use the variance of the time *k* parts depart to determine the *k*-lag covariance

$$Var(\Gamma_k) = \sum_{i=1}^{k}\sum_{j=1}^{k} Cov(T_i, T_j) = \sum_{i=1}^{k} Var(T_i) + 2\sum_{i=1}^{k}\sum_{j=i+1}^{k} Cov(T_i, T_j)$$

$$Var(\Gamma_k) = kVar(T) + 2\sum_{j=1}^{k-1}(k-j)Cov(T_1, T_{j+1})$$

Calculate — $Var(\Gamma_k)$

Determine — $Cov(T_1, T_{j+1})$

Departure Autocorrelation

for Production Systems can be determined analytically by

Extending this approach to analyze
sequence of *k* inter-departure times

$$Q^{(k)} = \left[\begin{array}{cccccc|c} Q_d & R_d & 0 & 0 & 0 & \dots & 0 \\ 0 & Q_d & R_d & 0 & 0 & \dots & 0 \\ 0 & 0 & Q_d & R_d & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & \dots & \dots & \dots & Q_d & R_d \\ \hline 0 & \dots & \dots & \dots & \dots & 0 & 0 \end{array}\right]$$

$$Q^{(k)} = \left[\begin{array}{c|c} Q_d^{(k)} & R_d^{(k)} \\ \hline 0 & 0 \end{array}\right] \longrightarrow Var(\Gamma_k)$$

Departure Autocorrelation
can be determined iteratively from the rate matrix

Proposition 2 *Starting with* $Var[\Gamma_1] = Var[T]$, $\Upsilon_1 = \Psi_1 = \Phi = Q_d^{-1}$ *and* $\Omega = -R_d\Phi$, *the following equations are solved for* $k = 2, 3, ..., m$ *iteratively to determine the covariance between the departure times with a lag of* $1, 2, \ldots, m-1$ *periods,* $Corr(T)_1, \ldots, Corr(T)_{m-1}$*:*

$$\Upsilon_k = \Upsilon_{k-1}\Omega \tag{23}$$

$$\Psi_k = \Psi_{k-1} + \Upsilon_k$$

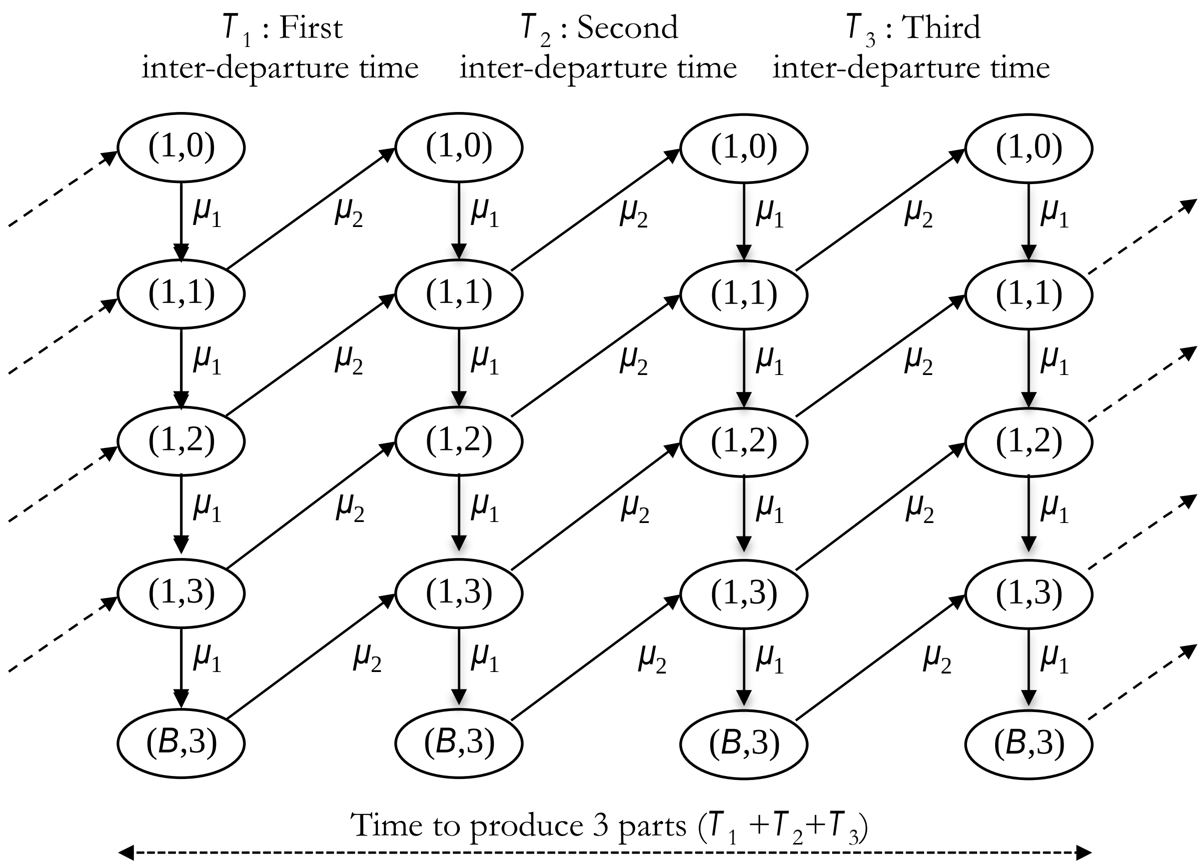
$$Var\left(\Gamma_k\right) = Var\left(\Gamma_{k-1}\right) + (1-2k)(E(T))^2 + 2\pi^{\text{entry}}\Upsilon_k\Psi_k u$$

$$Cov(T_1, T_k) = \begin{cases} \frac{1}{2}\left(Var\left(\Gamma_k\right) - kVar(T)\right) & \text{for k} = 2 \\ \frac{1}{2}\left(Var\left(\Gamma_k\right) - kVar(T) - 2\sum_{j=1}^{k-2}(k-j)Cov(T_1, T_{j+1})\right) & \text{for k} \geq 3 \end{cases}$$

$$Corr(T)_{k-1} = \frac{Cov(T_1, T_k)}{Var(T)}.$$

(Tan and Lagershausen 2017)

Example: Two-station Production Line
Exponential Processing Time and a Finite Buffer



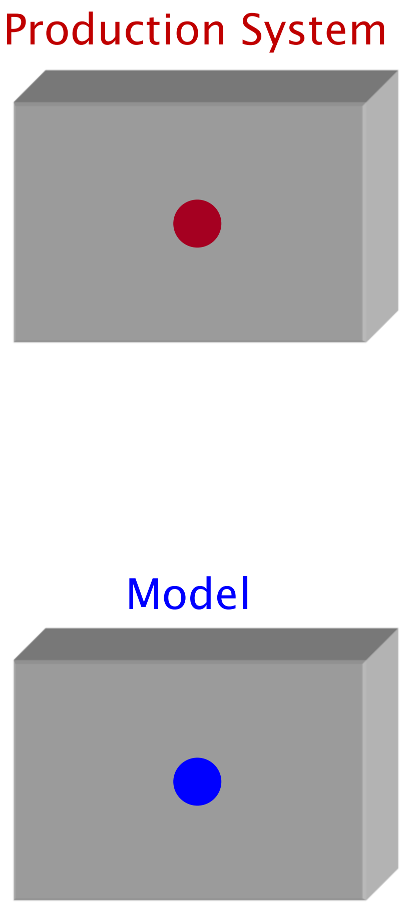
$$E(T) = \frac{(M+3)}{\mu(M+2)},$$

$$Var(T) = \frac{(M^2+6M+7)}{\mu^2(M+2)^2}$$

$$Corr(T)_1 = -\frac{1}{(M^2+6M+7)}$$

$$Corr(T)_2 = \frac{Cov(T_1,T_3)}{Var(T)} = -\frac{1}{(M^2+6M+7)}$$

Research Questions
Modelling and Analysis



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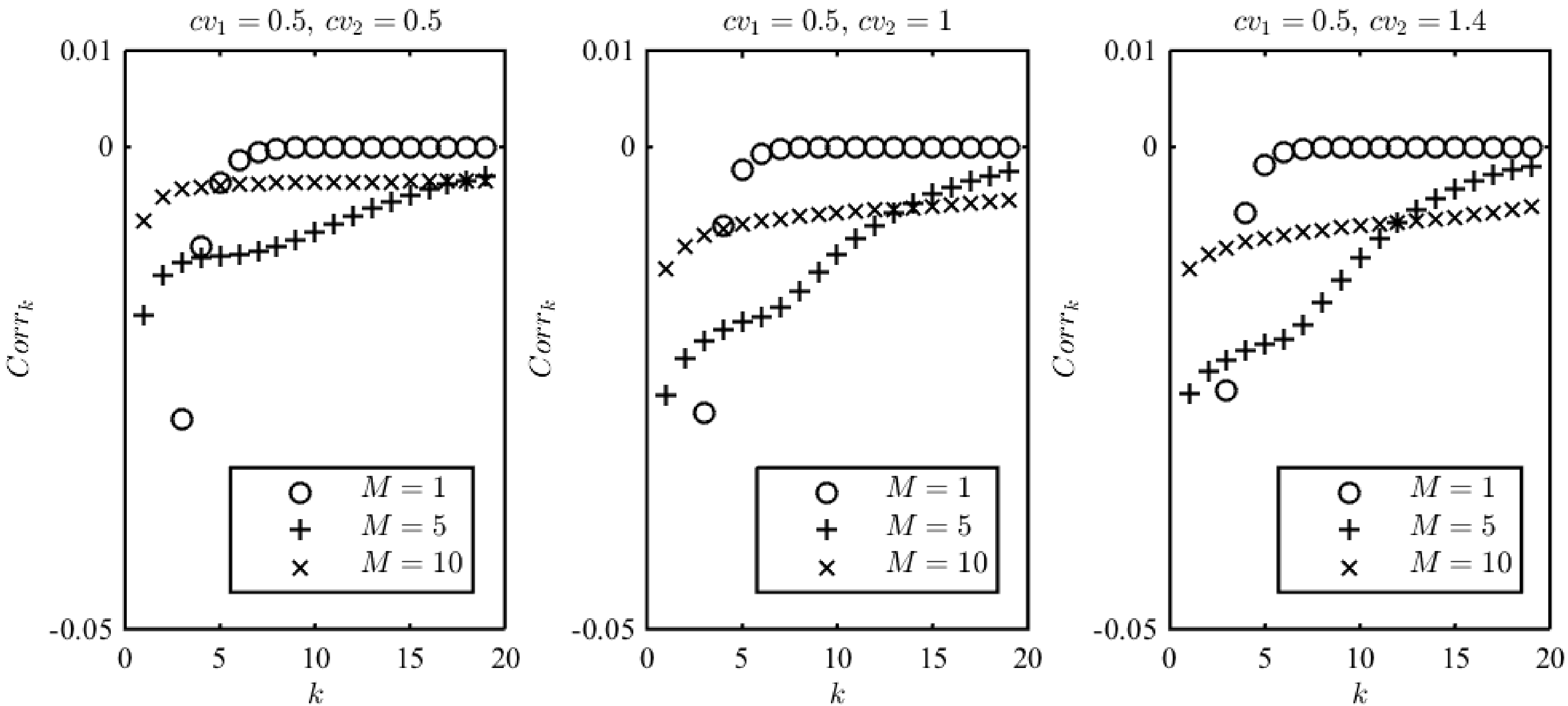
Correlation of Inter-departure Times

Two- Station Line with Coxian Service Time Distribution and a Finite Buffer $(/_1 = 1,\ /_2 = 1)$



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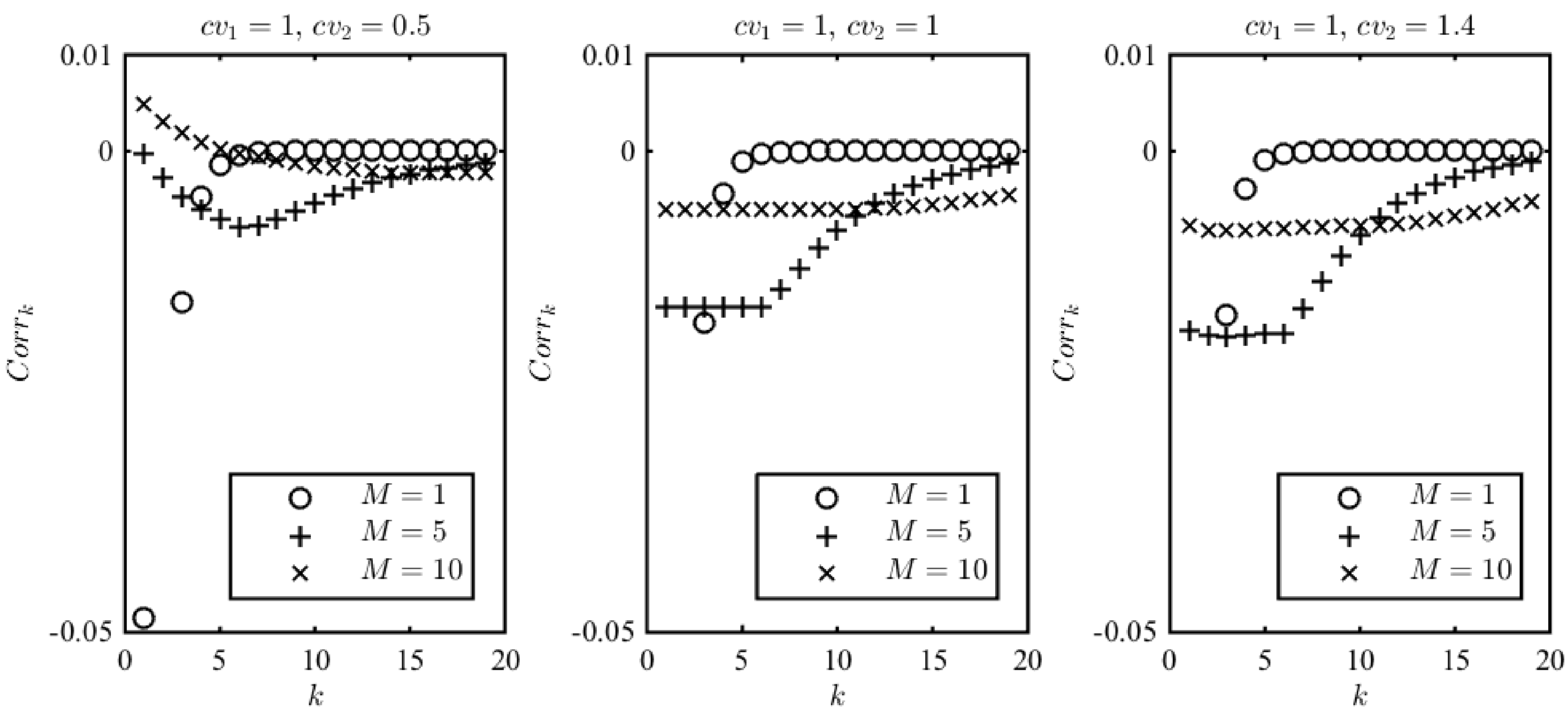
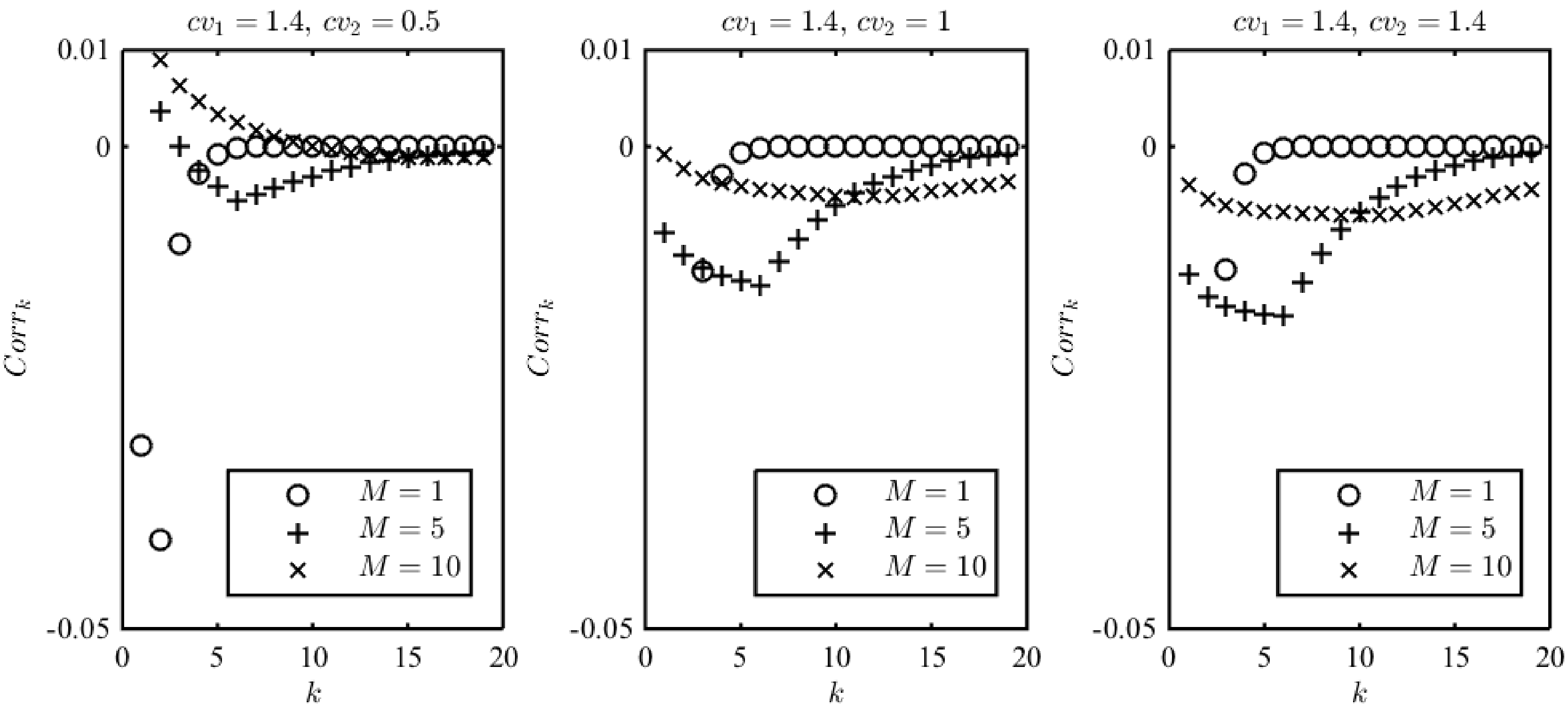
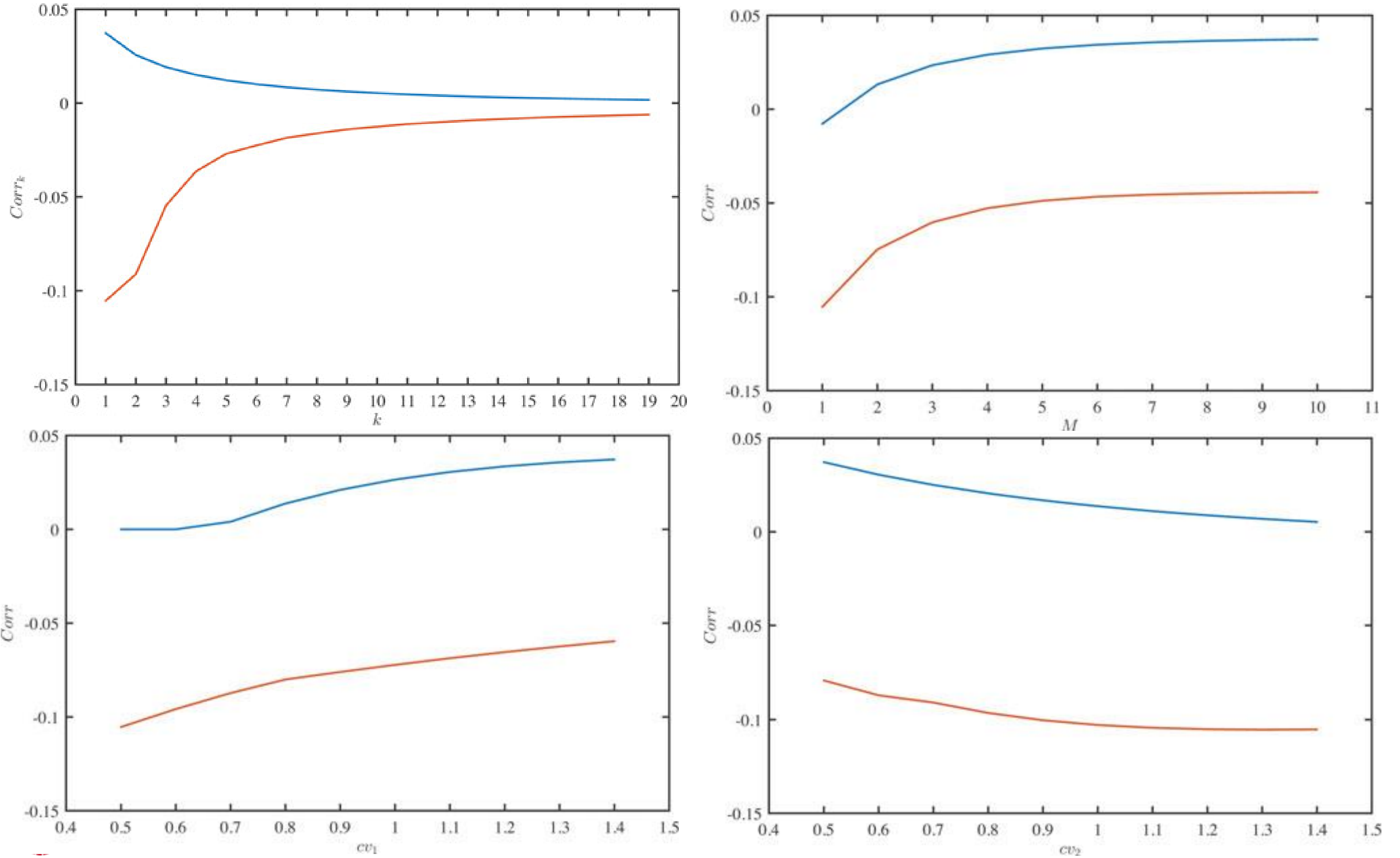


Correlation of Inter-departure Times

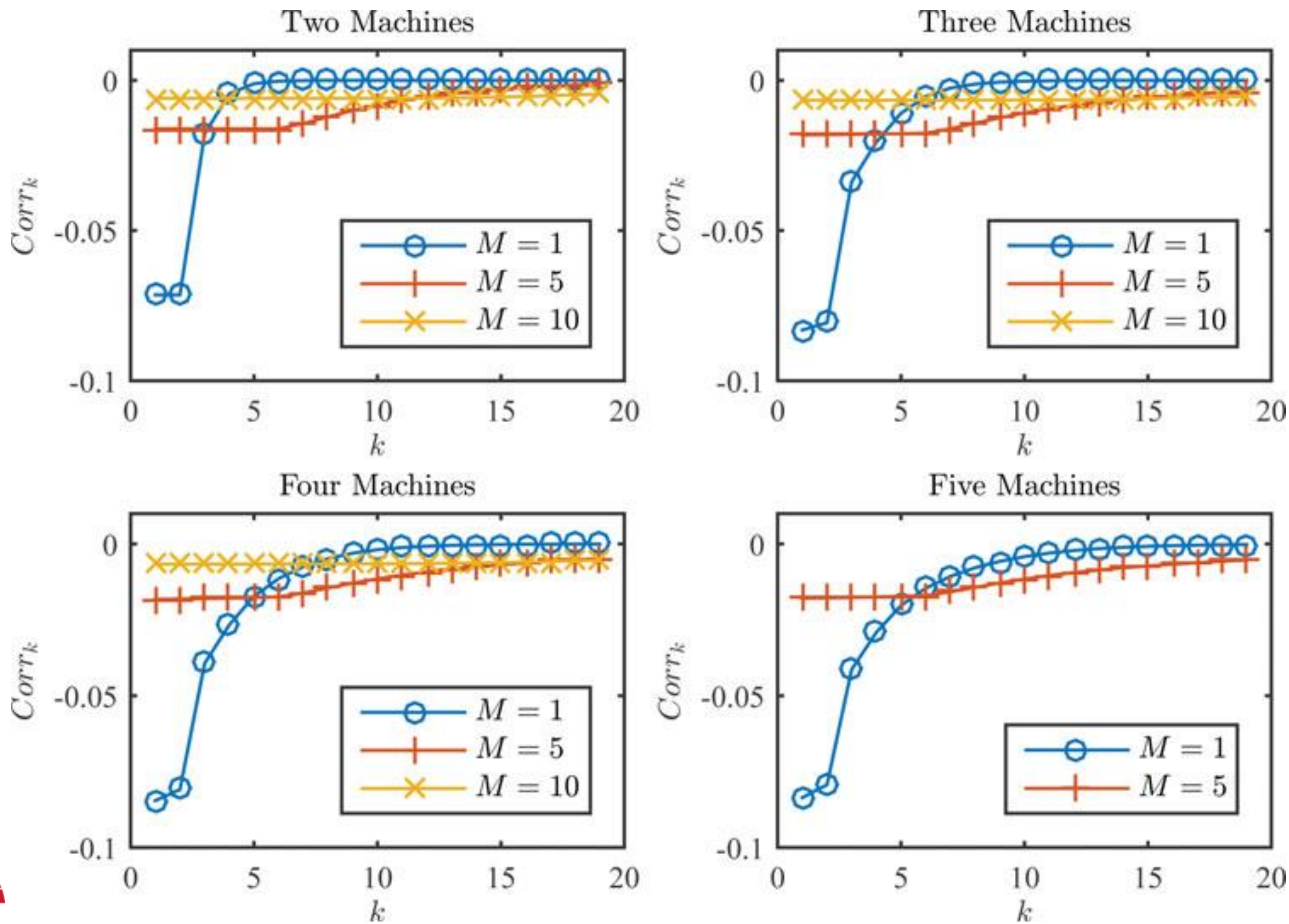
Two- Station Line with Coxian Service Time Distribution and a Finite Buffer ($/_1 = 1$, $/_2 = 1$)



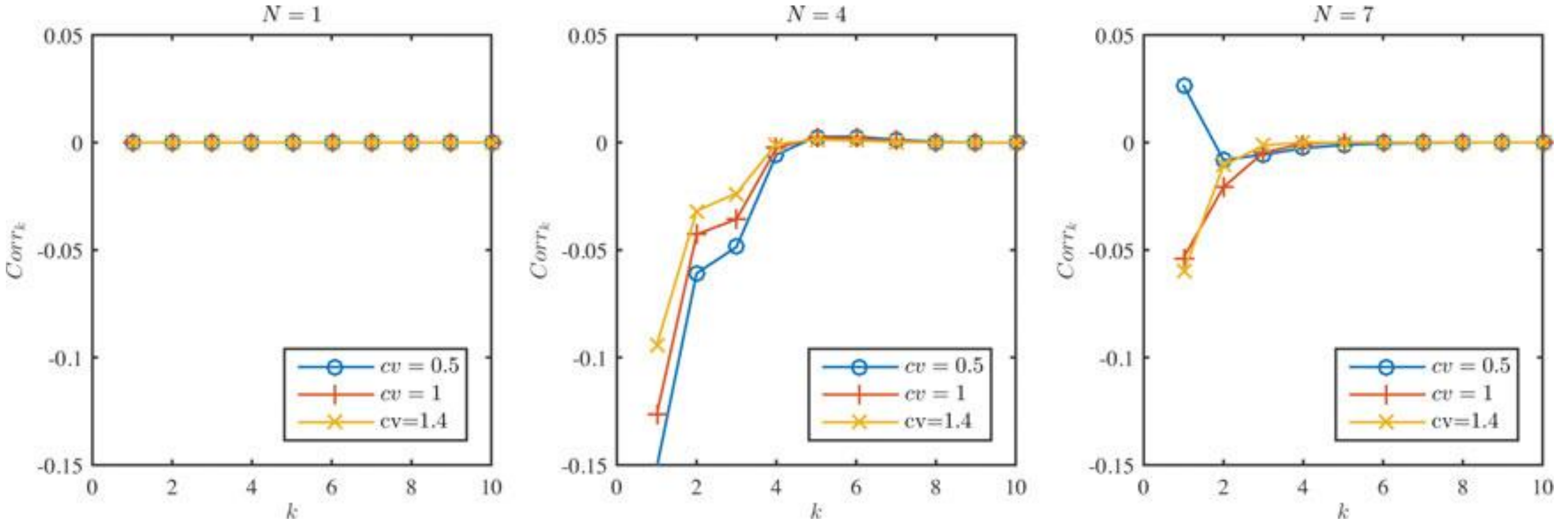
Two- Station Production Line with Coxian Processing Time and a Finite Buffer



Example: Multi-station Production Lines with Exponential Servers and Finite Buffers



Example: Closed Four-Station System with Different Service Time Coefficient of Variations and Number of Pallets



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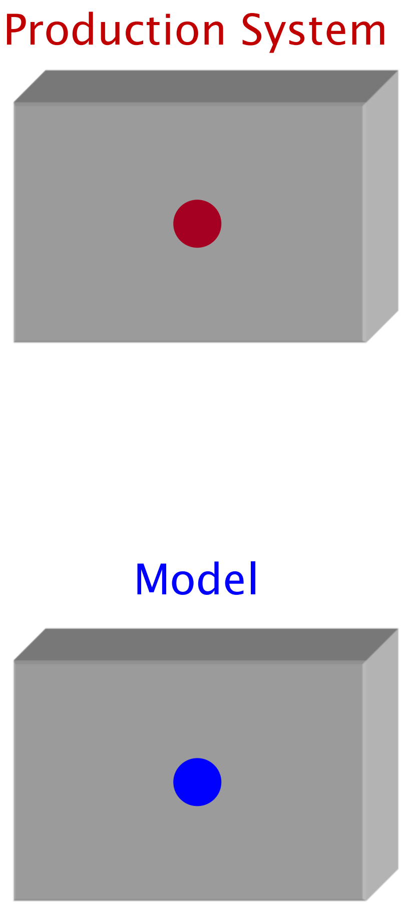
Modelling and Analysis



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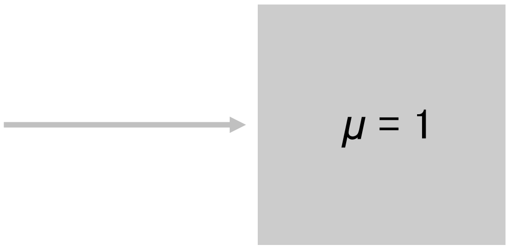
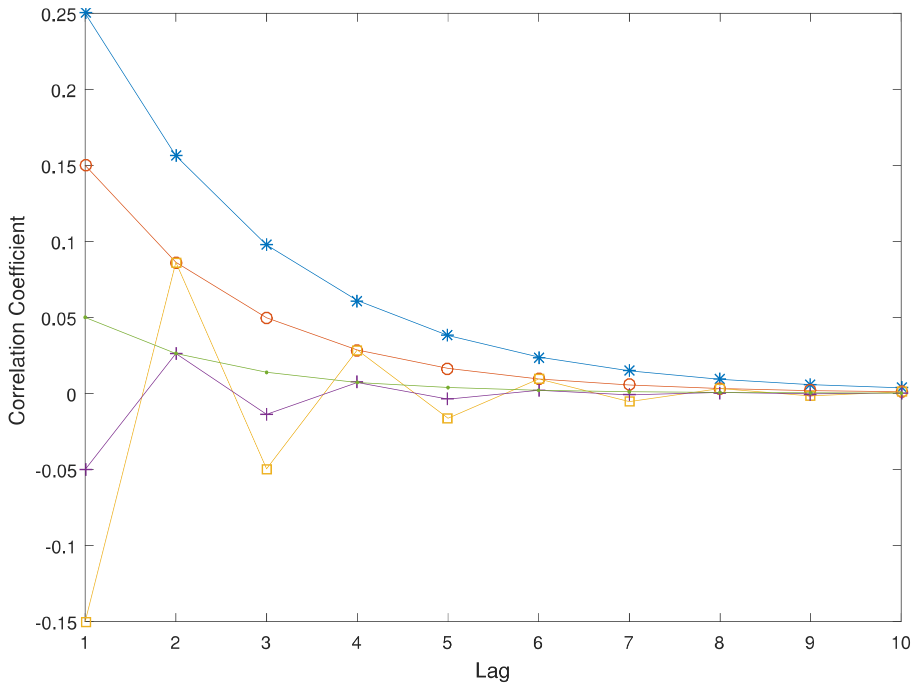
Exp.1:
An Infinite Buffer Queue with Correlated Arrival and Service Processes

Arrival

Service

$\lambda = 0.8$



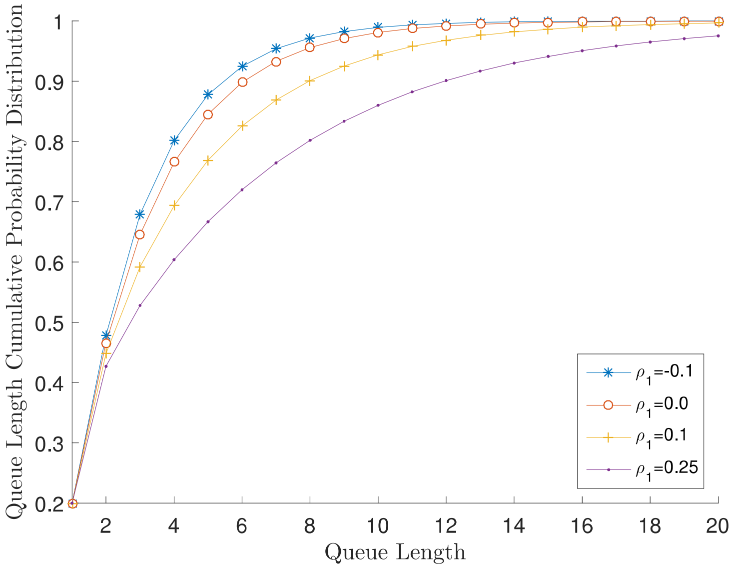


Correlated Arrival	Deterministic Service	MAP/D/1
Correlated Arrival	Renewal Service	MAP/PH/1
Renewal Arrival	Correlated Service	PH/MAP/1

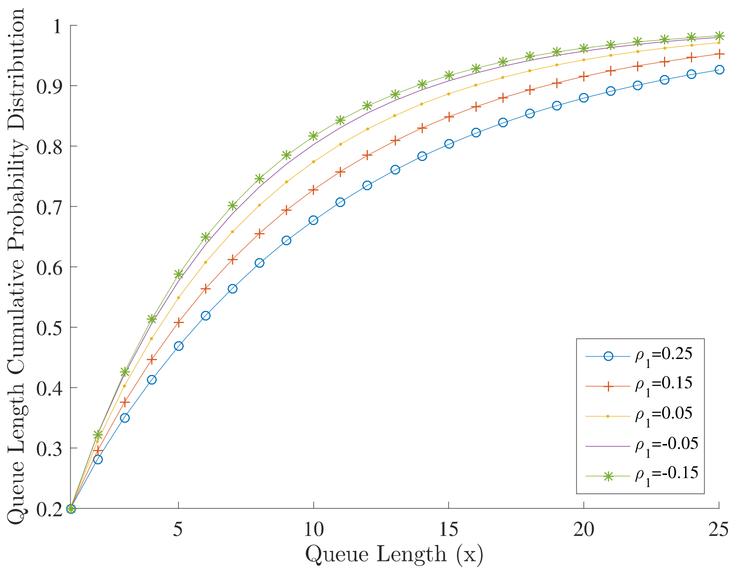
(Dizbin, 2016)

Structural Results for Processes with Positive and Negative Autocorrelations

MAP/D/1

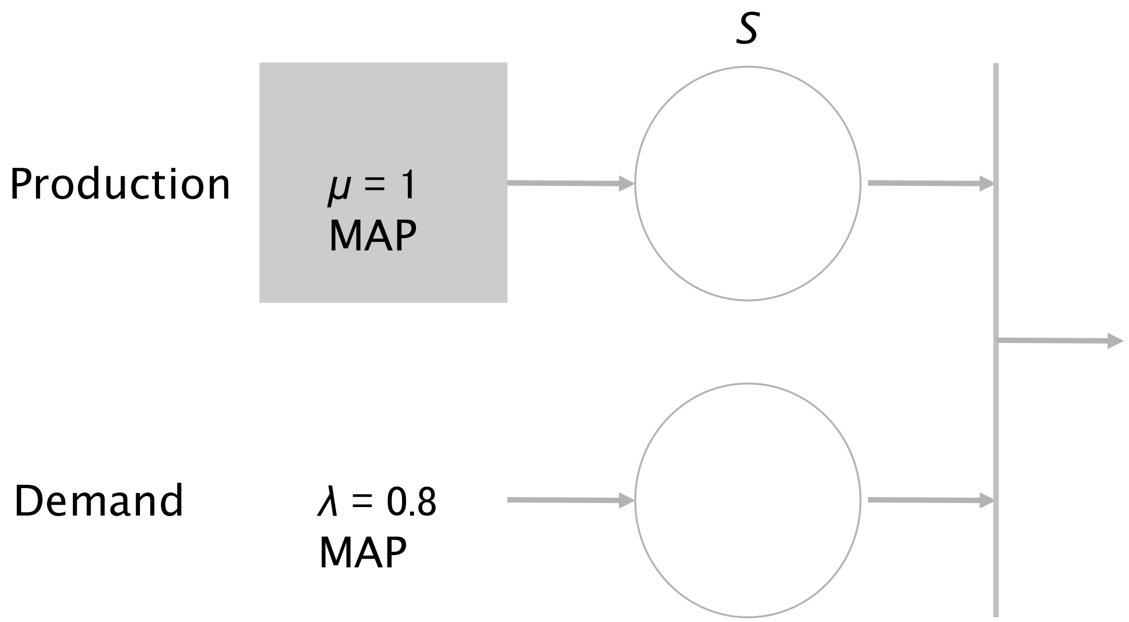


PH/MAP/1



- Positive Autocorrelation increases the probability of having **Higher Queue Lengths**
- Negative Autocorrelation increases the probability of having **Lower Queue Lengths**
- A process with negative autocorrelation stochastically **dominates** the same process with positive autocorrelation
- Expected number of customers and expected waiting time has an increasing convex order w.r.t autocorrelation

Exp. 2:
Base Stock System



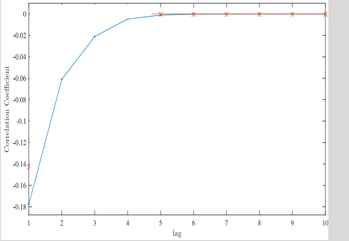
one, two-moment and renewal **approximations**

- **Exponential**
- **Coxian ($C_{2:b}$)**
- **Phase-type (PH)**

performance measures

- Base Stock Level
- Total Cost
- Expected Backlog
- Expected Inventory

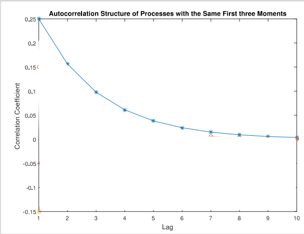
Base Stock Model with a Negatively Autocorrelated Process



$\rho = 0.8$

	S^*	TC	Error	$E[B]$	Error	$E[I]$	Error	$P(I<0)$	Error
MAP/MAP/1	4	3.1683		0.2079		2.1287		0.0990	
M/M/1	8	6.0128	90%	5.9361	179%	0.0154	-93%	0.0074	-93%
$C_{2:b}$/M/1	7	5.0979	61%	4.9503	133%	0.0295	-86%	0.0142	-86%
PH/M/1	7	5.0979	61%	4.9503	133%	0.0295	-86%	0.0142	-86%
MAP/M/1	5	3.5742	13%	3.0296	42%	0.1089	-48%	0.0521	-47%
M/$C_{2:b}$/1	7	5.0979	61%	4.9503	133%	0.0295	-86%	0.0142	-86%
$C_{2:b}$/$C_{2:b}$/1	6	4.2614	35%	3.9775	87%	0.0568	-73%	0.0272	-72%
PH/$C_{2:b}$/1	6	4.2614	35%	3.9775	87%	0.0568	-73%	0.0272	-72%
MAP/$C_{2:b}$/1	5	3.5742	13%	3.0296	42%	0.1089	-48%	0.0521	-47%
M/PH/1	7	5.0979	61%	4.9503	133%	0.0295	-86%	0.0142	-86%
$C_{2:b}$/PH/1	6	4.2614	35%	3.9775	87%	0.0568	-73%	0.0272	-72%
PH/PH/1	6	4.2614	35%	3.9775	87%	0.0568	-73%	0.0272	-72%
MAP/PH/1	4	3.1683		2.1287		0.2079		0.0990	0
M/MAP/1	6	4.2614	35%	3.9775	87%	0.0568	-73%	0.0272	-72%
$C_{2:b}$/MAP/1	5	3.5742	13%	3.0296	42%	0.1089	-48%	0.0521	-47%
PH/MAP/1	5	3.5742	13%	3.0296	42%	0.1089	-48%	0.0521	-47%

Base Stock Model with a Positively Autocorrelated Process

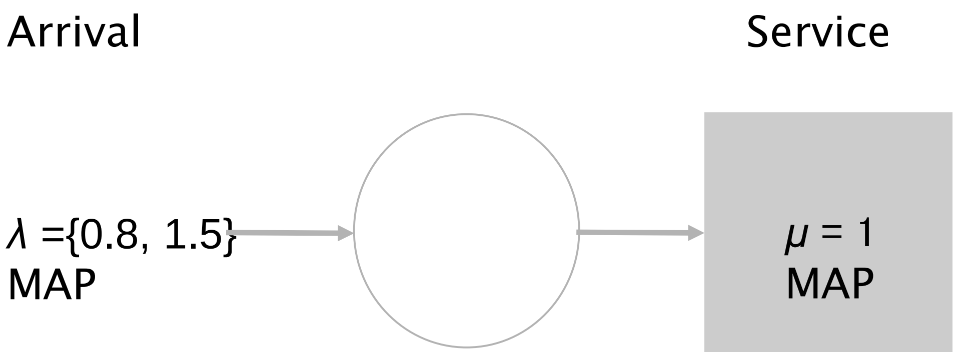


$\rho = 0.8$

	S^*	TC	Error	$E[B]$	Error	$E[I]$	Error	$P(I<0)$	Error
MAP/MAP/1	13	15.0901		1.4604		7.7880		0.1594	
M/M/1	8	16.9484	12%	3.9310	-50%	2.6035	78%	0.2843	78%
$C_{2:b}$/M/1	7	17.8635	18%	3.2502	-58%	2.9227	100%	0.3192	100%
PH/M/1	7	17.8635	18%	3.2502	-58%	2.9227	100%	0.3192	100%
MAP/M/1	11	15.3696	2%	6.1679	-21%	1.8403	26%	0.2009	26%
M/$C_{2:b}$/1	7	17.8635	18%	3.2502	-58%	2.9227	100%	0.3192	100%
$C_{2:b}$/$C_{2:b}$/1	6	19.0137	26%	2.6086	-67%	3.2810	125%	0.3584	125%
PH/$C_{2:b}$/1	7	17.8635	18%	3.2502	-58%	2.9227	100%	0.3192	100%
MAP/$C_{2:b}$/1	11	15.3696	2%	6.1679	-21%	1.8403	26%	0.2009	26%
M/PH/1	7	17.8635	18%	3.2502	-58%	2.9227	100%	0.3192	100%
$C_{2:b}$/PH/1	6	19.0137	26%	2.6086	-67%	3.2810	125%	0.3584	125%
PH/PH/1	6	19.0137	26%	2.6086	-67%	3.2810	125%	0.3584	125%
MAP/PH/1	11	15.3696	2%	6.1679	-21%	1.8403	26%	0.2009	26%
M/MAP/1	9	16.2425	8%	4.6467	-40%	2.3192	59%	0.2532	59%
$C_{2:b}$/MAP/1	8	16.9484	12%	3.9310	-50%	2.6035	78%	0.2843	78%
PH/MAP/1	9	16.2425	8%	4.6467	-40%	2.3192	59%	0.2532	59%

Renewal approximation yields significant errors

Exp. 3:
A Finite Buffer Queue



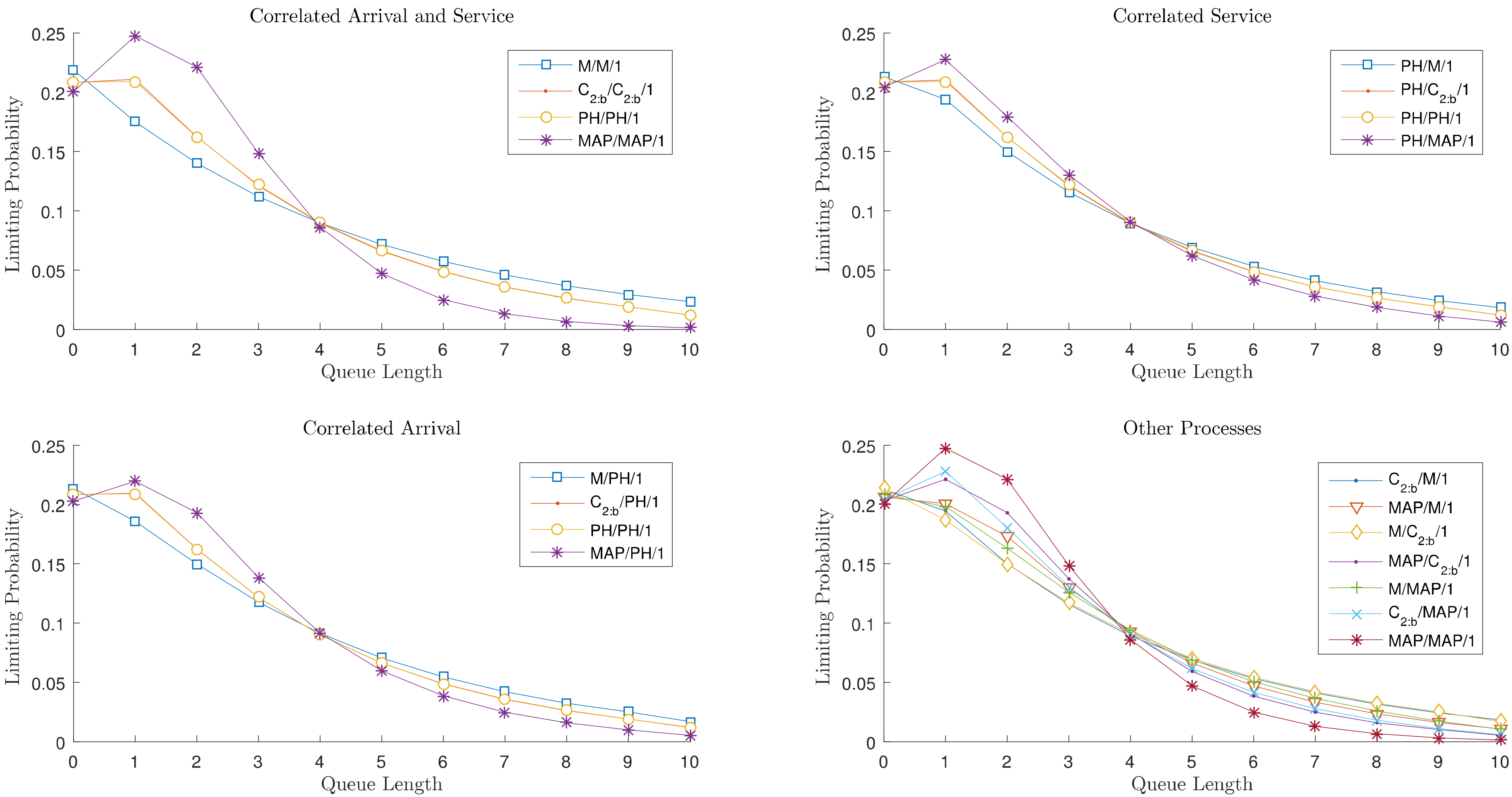
one, two-moment and renewal **approximations**

- **Exponential**
- **Coxian ($C_{2:b}$)**
- **Phase-type (PH)**

performance measures

- blocking probability
- idle probability
- expected number of customers
- expected waiting time

Impact of Negative Autocorrelation on the Probability Distribution of the Number of Customers

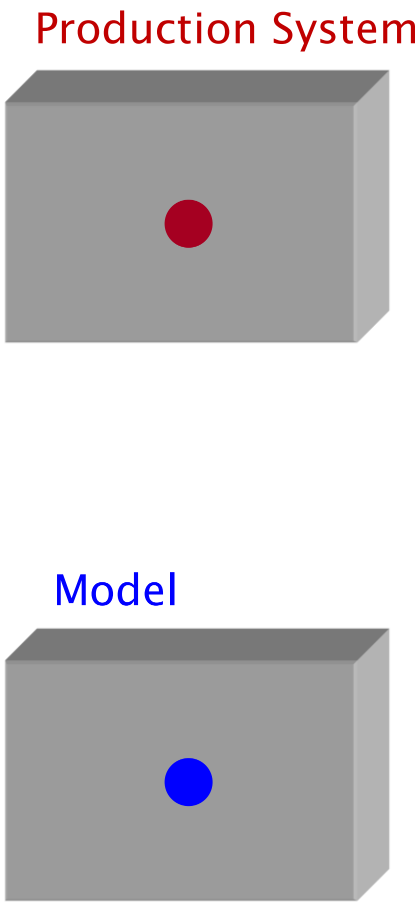


K=9, ρ=0.8

Effect of Renewal Estimation of the Correlated Processes

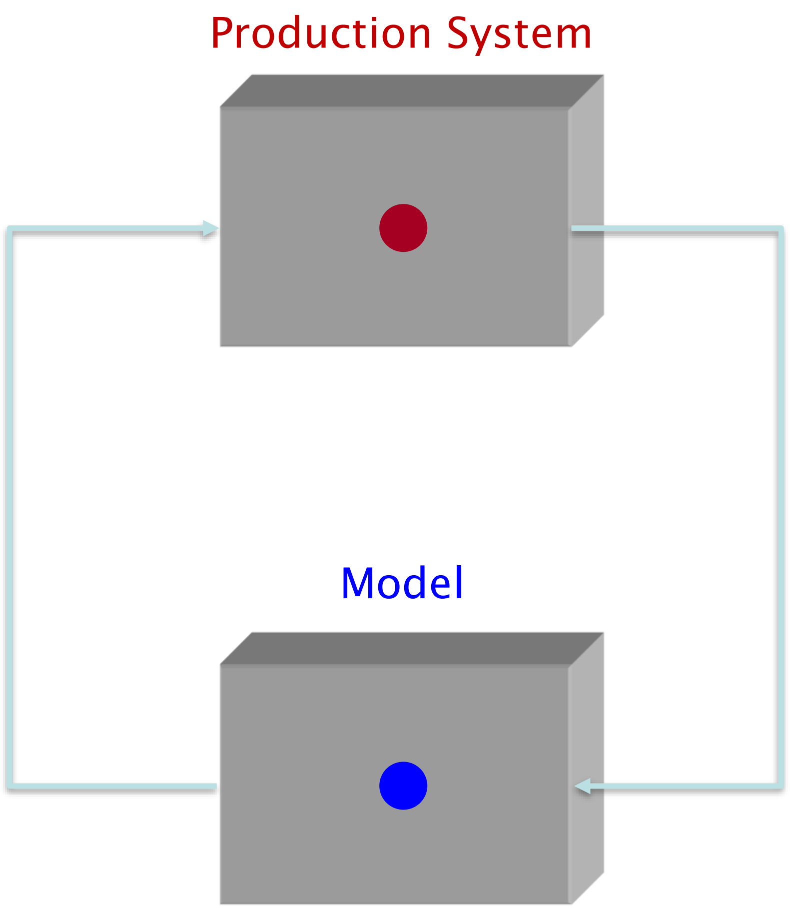
		Correlated Arrival		Correlated Service	
		$\rho = 0.8$	$\rho = 1.5$	$\rho = 0.8$	$\rho = 1.5$
- Corr	π_0	+	+	+	+
	B	+	-	+	=
	L_s	+	-	+	-
	W_s	+	-	+	-
+ Corr	π_0	-	-	-	-
	B	-	+	-	-
	L_s	-	+	-	+
	W_s	-	+	-	+

Research Questions
Modelling and Analysis



- Capturing **output dynamics** from production systems accurately in analytical models
- **Analytical methods** to analyze the statistical properties of output dynamics from production systems
- Understanding **the effects** of system parameters on the output dynamics
- Understanding **the impact** of autocorrelation on the performance of production systems

Research Questions
Data-Driven Modelling and Control



- **Fitting** models based on observed **data**
- **Controlling** production systems effectively by using models that capture output dynamics
- Controlling production systems by **modulating** the input arrival streams

There are numerous models to fit a MAP based on observed data

PH Fitting Methods

- Hyper-Erlang Fitting
- Moment Fitting

MAP Fitting Methods

- Expectation Maximization (EM)
- Two-Stage EM algorithm
- Autocorrelation Fitting
- Joint Moment Fitting
- Markov Exponential Process
- Kronecker Product Composition

Comparison of Different Estimation Methods

Paper	Fitting Approach	Input Data	Output Process	Algorithm Specification
[Horvath et al., 2005]	Moment Based	Inter-Event Time	MAP	Autocorrelation Fitting
[Buchholz and Kriege, 2009]	Moment Based	Inter-Event Time	MAP	Joint Moment Fitting
[Casale et al., 2010]	Moment Based	Inter-Event Time	MAP	Kronecker Product Composition
[Telek and Horvath, 2007]	Moment Based	Inter-Event Time	MAP	Minimal Representation
[Horvath, 2013]	Moment Based	Inter-Event Time	MAP	-
[Buchholz, 2003]	Maximum Likelihood	Inter-Event Time	MAP	Uniformization Technique
[Breuer, 2002]	Maximum Likelihood	Inter-Event Time	Batch MAP	Uniformization Technique
[Klemm et al., 2003]	Maximum Likelihood	Inter-Event Time	Batch MAP	-
[Kriege and Buchholz, 2014]	Maximum Likelihood	Inter-Event Time	MAP	Aggregated Data
[Buchholz and Panchenko, 2004]	Maximum Likelihood	Inter-Event Time	MAP	Faster Convergence
[Okamura et al., 2013]	Maximum Likelihood	Inter-Event Time	MAP	Deterministic Annealing
[Horvath and Okamura, 2013]	Maximum Likelihood	Inter-Event Time	Marked MAP	-
[Okamura et al., 2009]	Maximum Likelihood	Group Data	MAP	-
[Breuer and Kume, 2010]	Maximum Likelihood	Group Data	MAP	-
[Buchholz et al., 2010]	Moment Based	Inter-Event Time	Marked MAP	-

Estimation Methods work well

for systems with non-negative auto-correlation with lots of data

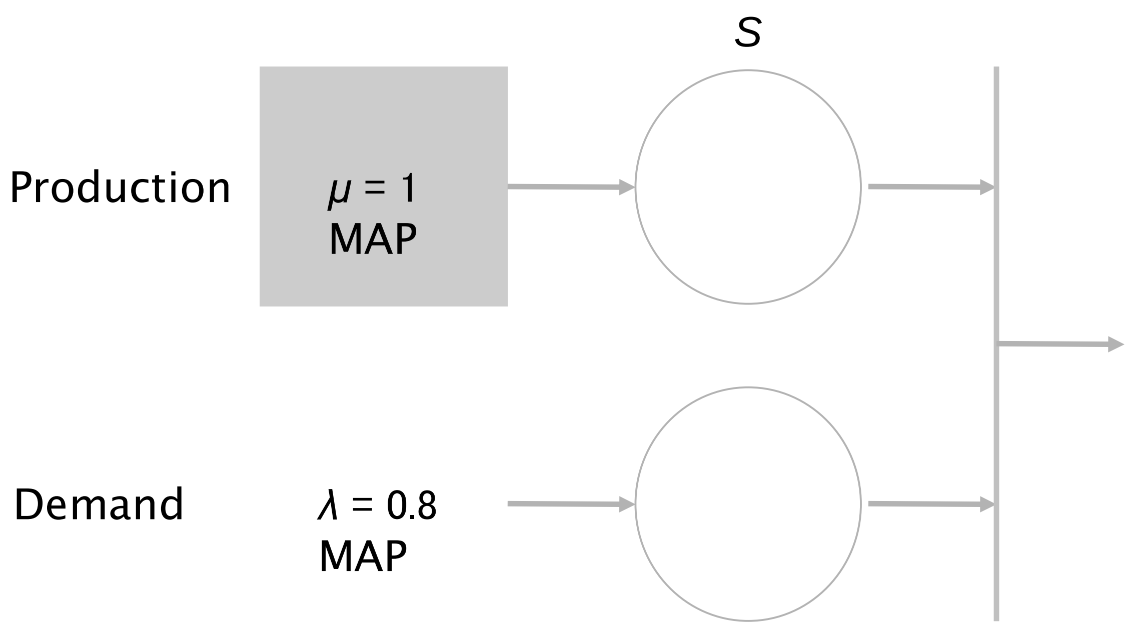
	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	mom1	mom2	mom3
Inp	-0.2891	0.2498	-0.2159	0.1866	-0.1613	1.0000	3.7490	21.5594
Inp1-1m	-0.2895	0.2519	-0.2164	0.1860	-0.1619	1.0021	3.7666	21.6711
MOEA3	-0.2776	0.2308	-0.1919	0.1595	-0.1327	1.2388	4.3066	23.0757
MOEA6	-0.2821	0.2491	-0.2194	0.1934	-0.1704	1.5159	7.6793	62.2628
MOEA10	-0.1649	0.1049	-0.0680	0.0437	-0.0281	1.1679	4.7076	30.5565
Mfit10-JMom	0.0000	0.0000	0.0000	0.0000	0.0000	1.0021	3.7666	21.6703
Mfit10-AC10	0.0000	0.0000	0.0000	0.0000	0.0000	1.0021	3.7667	21.6722
Mfit3-JMom	-0.2636	0.2238	-0.1900	0.1613	-0.1369	1.0021	3.7666	21.6711
Mfit3-AC10	-0.2322	0.2318	-0.1914	0.1767	-0.1526	1.0021	3.7666	21.6713
Mfit10-ACfit4	0.0000	0.0000	0.0000	0.0000	0.0000	1.0021	3.7667	21.6722
Mfit3-ACfit4	-0.2273	0.2414	-0.1884	0.1863	-0.1534	1.0021	3.7666	21.6712
Mfit6-ACfit4	0.0000	0.0000	0.0000	0.0000	0.0000	1.0021	3.7666	21.6711
Mfit6-JMom	0.0000	0.0000	0.0000	0.0000	0.0000	1.0021	3.7666	21.6711
Mfit6-ACfit10	0.0000	0.0000	0.0000	0.0000	0.0000	1.0021	3.7666	21.6711
Gfit10-JMom	-0.1754	0.0981	-0.0549	0.0307	-0.0172	1.0021	3.7644	21.6316
Gfit10-AC10	-0.2727	0.2393	-0.2090	0.1826	-0.1596	1.0021	3.7645	21.6316
Gfit3-JMom	-0.2689	0.2277	-0.1928	0.1632	-0.1382	1.0021	3.7697	21.7738
Gfit3-AC10	-0.2840	0.2469	-0.2140	0.1855	-0.1607	1.0021	3.7697	21.7738
Gfit10-ACfit4	-0.2737	0.2440	-0.2164	0.1920	-0.1703	1.0021	3.7644	21.6316
Gfit3-ACfit4	-0.2851	0.2487	-0.2164	0.1882	-0.1637	1.0021	3.7697	21.7738
Gfit6-ACfit4	-0.2801	0.2462	-0.2164	0.1902	-0.1672	1.0021	3.7715	21.7784
Gfit6-JMom	-0.2215	0.1547	-0.1081	0.0755	-0.0527	1.0021	3.7715	21.7784
Gfit6-ACfit10	-0.2777	0.2419	-0.2107	0.1836	-0.1599	1.0021	3.7715	21.7784

Estimation Methods do **not** work well

for systems with **negative** auto-correlation with lots of data

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	mom1	mom2	mom3
exp4	-0.1775	-0.0610	-0.0211	-0.0048	-0.0011	1.0000	1.7254	4.0883
exp4-1m	-0.1774	-0.0606	-0.0200	-0.0051	0.0000	1.0003	1.7277	4.0984
MOEA3	-0.0394	-0.0001	0.0000	0.0000	0.0000	1.0587	2.1300	6.2366
MOEA6	-0.0196	0.0000	0.0000	0.0000	0.0000	0.9063	1.5480	3.8800
MOEA10	-0.0133	0.0000	0.0000	0.0000	0.0000	0.9639	1.7727	4.8423
Mfit10-JMom	0.0000	0.0000	0.0000	0.0000	0.0000	1.0003	1.7277	4.0984
Mfit10-AC10	0.0000	0.0000	0.0000	0.0000	0.0000	1.0003	1.7277	4.0985
Mfit3-JMom	-0.0119	-0.0008	0.0000	0.0000	0.0000	1.0003	1.7276	4.0984
Mfit3-AC10	-0.0119	-0.0008	-0.0001	0.0000	0.0000	1.0003	1.7276	4.0984
Mfit10-ACfit4	0.0000	0.0000	0.0000	0.0000	0.0000	1.0003	1.7277	4.0985
Mfit3-ACfit4	-0.0119	-0.0008	-0.0001	0.0000	0.0000	1.0003	1.7276	4.0984
Mfit6-ACfit4	0.0000	0.0000	0.0000	0.0000	0.0000	1.0002	1.7277	4.0984
Mfit6-JMom	0.0000	0.0000	0.0000	0.0000	0.0000	1.0003	1.7277	4.0984
Mfit6-ACfit10	0.0000	0.0000	0.0000	0.0000	0.0000	1.0002	1.7277	4.0984
Gfit10-JMom	-0.0350	0.0001	0.0007	-0.0001	0.0000	1.0003	1.7311	4.1319
Gfit10-AC10	-0.1116	0.1130	-0.1105	0.1087	-0.1066	1.0003	1.7311	4.1320
Gfit3-JMom	-0.0759	0.0459	-0.0277	0.0168	-0.0101	1.0003	1.7465	4.2582
Gfit3-AC10	-0.1093	0.0957	-0.0838	0.0734	-0.0643	1.0003	1.7465	4.2582
Gfit10-ACfit4	-0.0642	0.0335	-0.0178	0.0095	-0.0051	1.0003	1.7311	4.1320
Gfit3-ACfit4	-0.0678	0.0368	-0.0200	0.0109	-0.0059	1.0003	1.7465	4.2582
Gfit6-ACfit4	-0.0802	0.0401	-0.0200	0.0100	-0.0050	1.0003	1.7287	4.1105
Gfit6-JMom	-0.1135	0.0900	-0.0720	0.0576	-0.0461	1.0003	1.7287	4.1105
Gfit6-ACfit10	-0.1184	0.1100	-0.1021	0.0949	-0.0881	1.0003	1.7287	4.1105

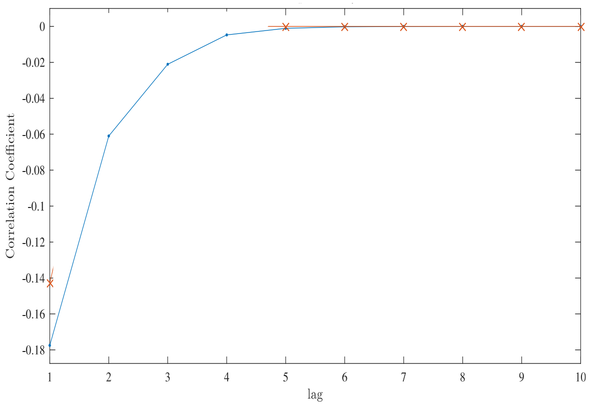
Base Stock System with MAP Estimation



Use the **estimated** arrival and service processes in order to determine the **base stock level**

Compare with the **optimal** performance

Base Stock System with MAP Estimation

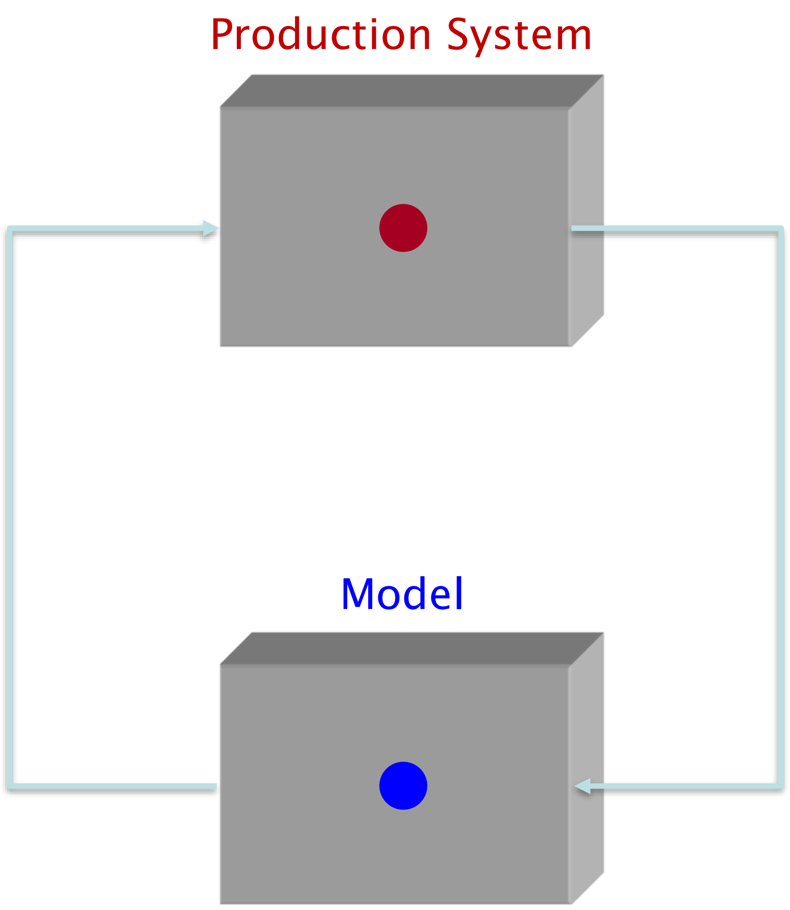


	S^*	TC	Error	$E[B]$	Error	$E[I]$	Error	$P(I<0)$	Error
MAP/MAP/1	4	3.1683		0.2079		2.1287		0.0990	
$M\hat{A}P/M\hat{A}P/1$	5	3.5742	13%	3.0296	42%	0.1089	-48%	0.0521	-47%
$M\hat{A}P$/MAP/1	4	3.1683		2.1287		0.2079		0.0990	
MAP/$M\hat{A}P/1$	4	3.1683		2.1287		0.2079		0.0990	
PH/PH/1	6	4.2614	35%	3.9775	87%	0.0568	-73%	0.0272	-72%

MAP fitting allows us to control production systems more effectively

with lots of data

Research Questions
Data-Driven Modelling and Control



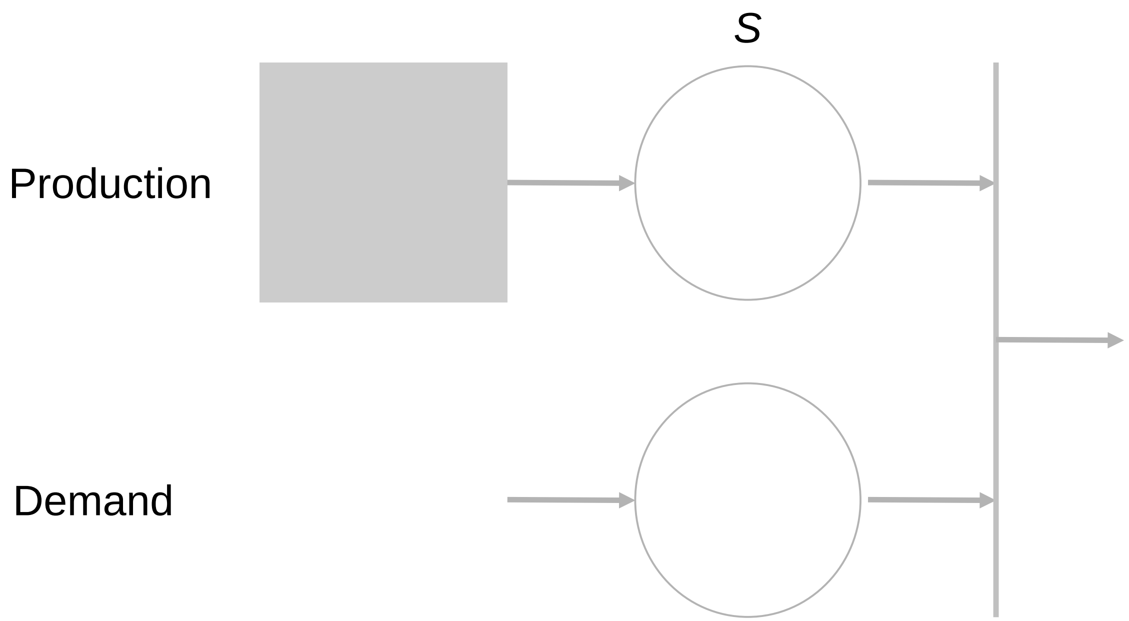
- **Fitting** models based on observed **data**
- **Controlling** production systems effectively by using models that capture output dynamics
- Controlling production systems by **modulating** the input arrival streams

Joint Simulation-Optimization Approach for Determining Optimal Base-stock Levels

- Use the **observed demand trace** as input
- Generate **replications** for the service time
- Generate **shortfall process** for each replication
- Solve an **optimization problem** to determine the optimal base-stock level for the given trace and replications

$$\min_{S \in \left\{0,1,\ldots,\max_{i,r}\{x_{i,r}\}\right\}} \quad C(S) = \sum_{r=1}^{R} \left(h \left(\sum_{i=1}^{n-1} \left(t_{i+1,r} - t_{i,r}\right) \left[S - x_{i,r}\right]^{+} \right) + b \left(\sum_{i=1}^{n-1} \left(t_{i+1,r} - t_{i,r}\right) \left[x_{i,r} - S\right]^{+} \right) \right)$$

Base Stock System with Observed Data



- Use data from a trace
- Find the base stock level by using math programming
- Estimate MAP and determine the base stock

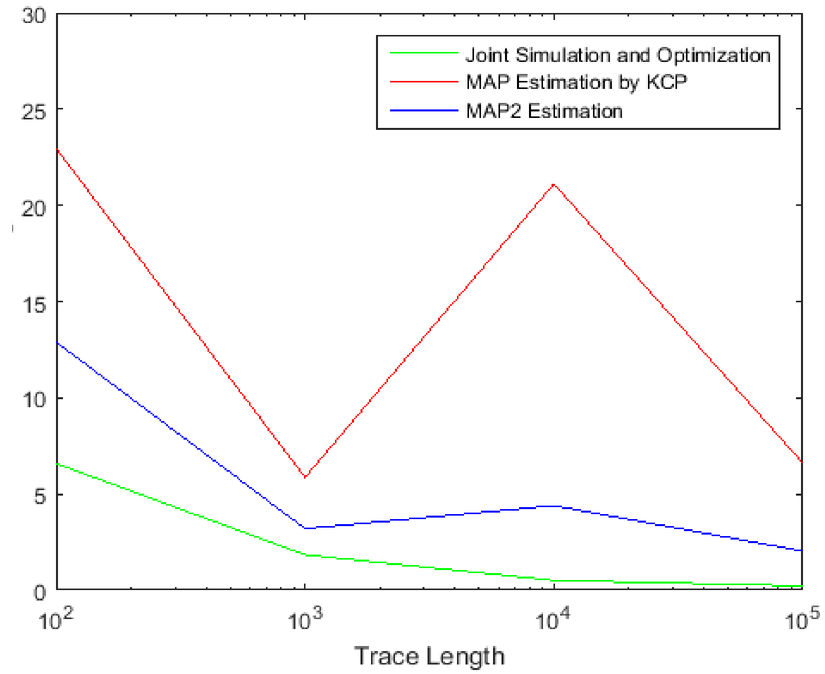
Compare with the **optimal** performance

(Khayyati and Tan 2017)

% of the cases where Joint Simulation and Optimization results are **as good as or better** than methods that use MAP fitting

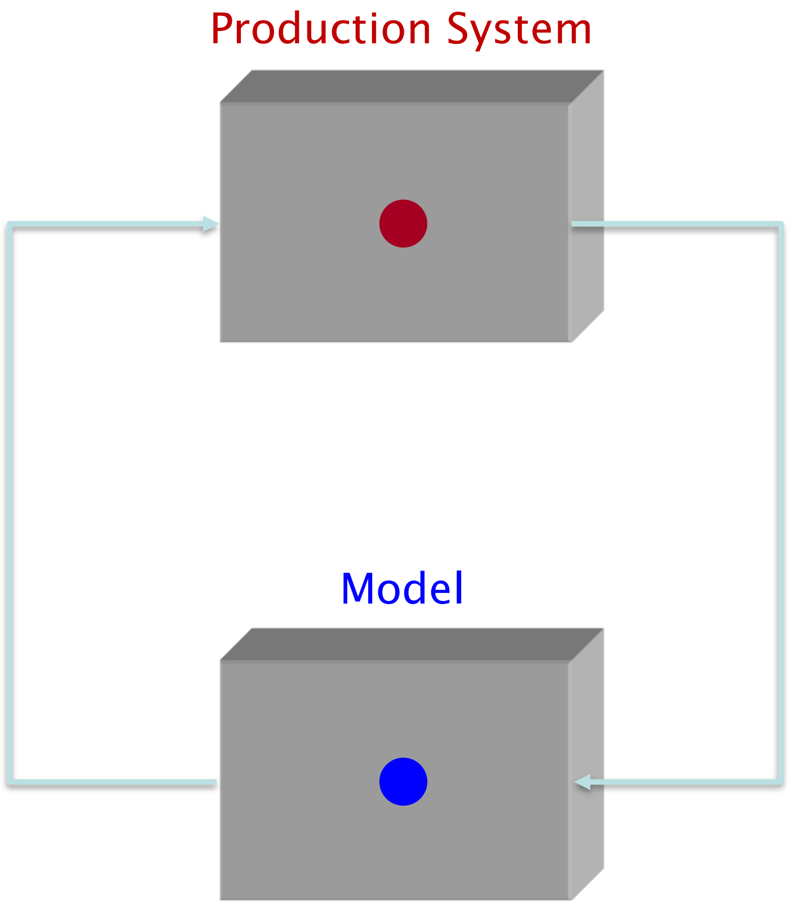
			Trace Length			
CV	Utilization	ρ_1	100	1000	10000	100000
0.59	0.4	0.3	100%	100%	100%	100%
		0	100%	100%	100%	100%
		-0.3	100%	100%	100%	100%
	0.6	0.3	78%	80%	100%	100%
		0	100%	100%	100%	100%
		-0.3	100%	100%	100%	100%
1.67	0.4	0.3	100%	76%	75%	98%
		0	100%	100%	100%	100%
		-0.3	75%	100%	100%	100%
	0.6	0.3	100%	48%	87%	100%
		0	100%	100%	100%	100%
		-0.3	78%	79%	100%	100%

% Deviation from the Optimal Cost



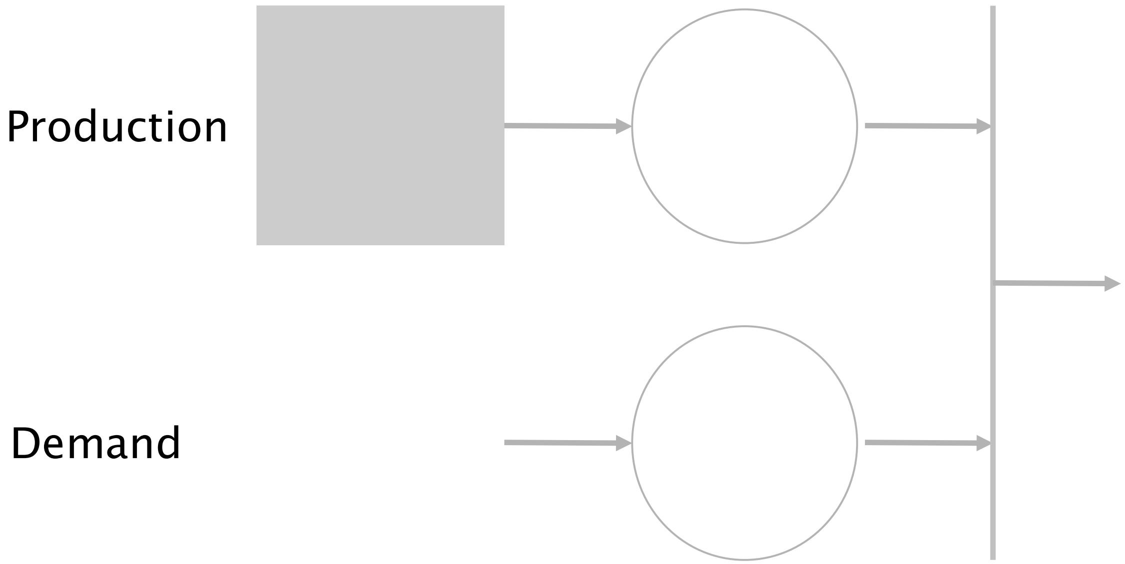
Inter-arrival time: MAP
Processing time: Exponential

Research Questions
Data-Driven Modelling and Control



- **Fitting** models based on observed **data**
- **Controlling** production systems effectively by using models that capture output dynamics
- Controlling production systems by **modulating** the input arrival streams

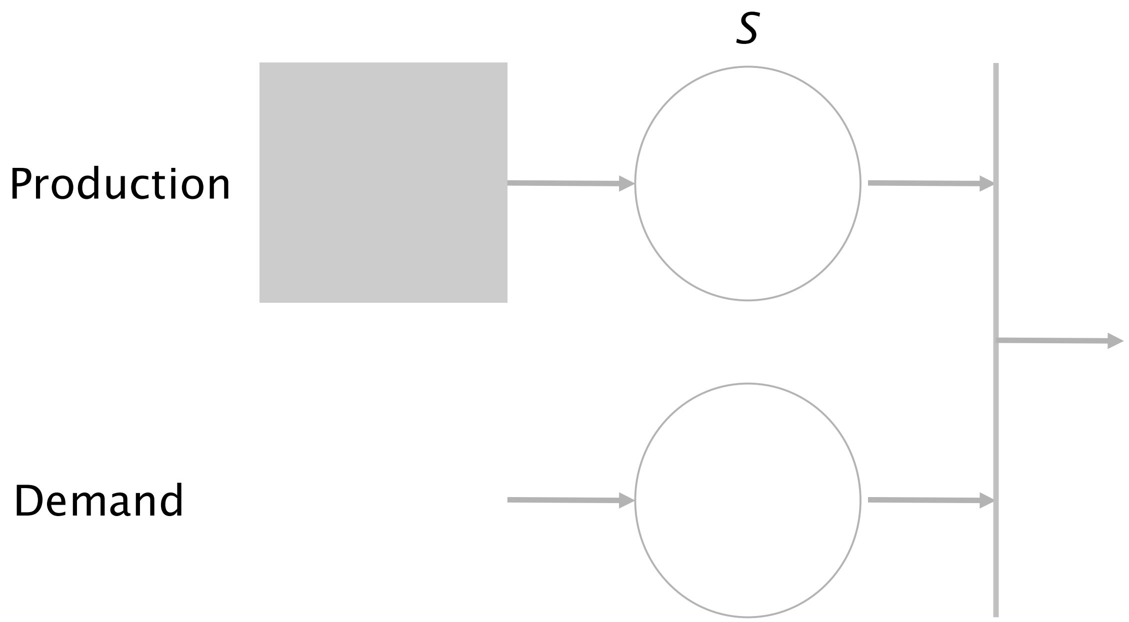
Optimal Production Policy for a System with Correlated Service and Demand



State-dependent base-stock policy
is optimal for the production and inventory control problem with MAP arrival and production processes.

(Dizbin and Tan, 2017)

State-dependent Base Stock System



- MAP Arrival
- Exponential Service

Compare Total Cost

- State-dependent Base Stock Policy
- Single Base Stock Policy

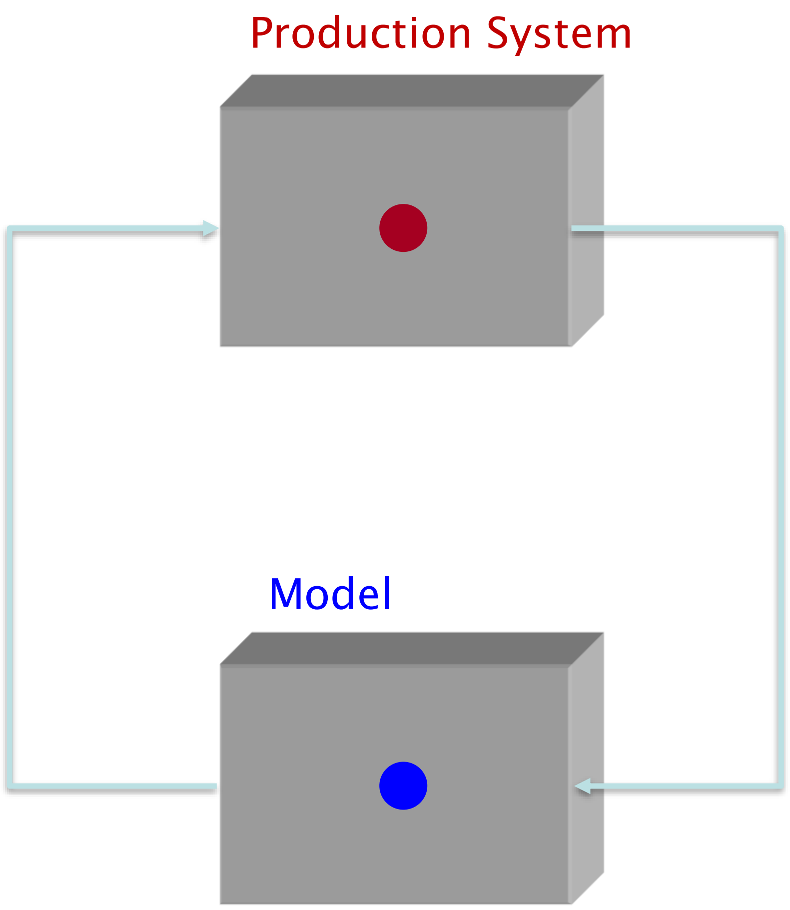
Control of a Production System with Positively Correlated Arrivals

		$\bar{Z}^*$	TC	Error	$E[B]$	Error	$E[I]$	Error	$Pr(I<0)$	Error
Opt	MAP/M/1	(11,12,19)	15.5317		1.6262		7.4006		0.1606	
App1	MAP/M/1	13	15.8697	2%	1.5373	-5%	8.1829	11%	0.1518	-5%
App2	PH/M/1	7	17.8717	15%	2.8710	77%	3.5166	-52%	0.2846	77%
App3	$C_{2:b}/M/1$	7	17.8717	15%	2.8710	77%	3.5166	-52%	0.2846	77%
App4	M/M/1	8	17.1643	11%	2.5864	59%	4.2320	-43%	0.2559	59%

$cv < 1$ $\rho = 0.8$

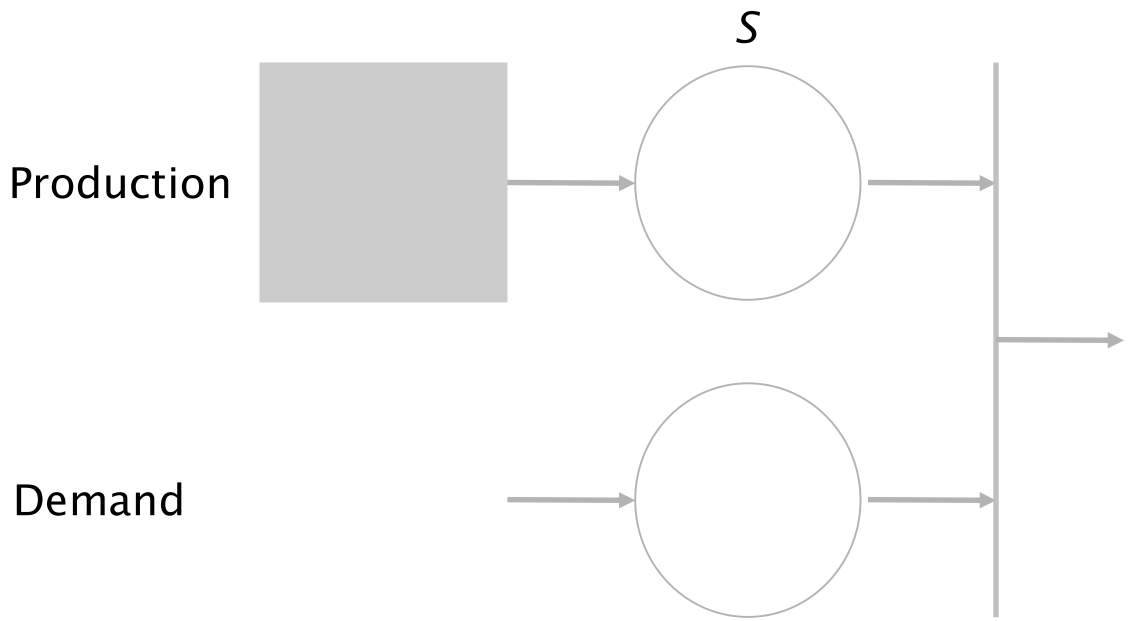
A state-dependent control of production systems is more effective

Research Questions
Data-Driven Modelling and Control



- **Fitting** models based on observed **data**
- **Controlling** production systems effectively by using models that capture output dynamics
  - How can we detect the state of the process from the data and control the system accordingly?
- Controlling production systems by **modulating** the input arrival streams

State-dependent Base Stock System with Observed Data

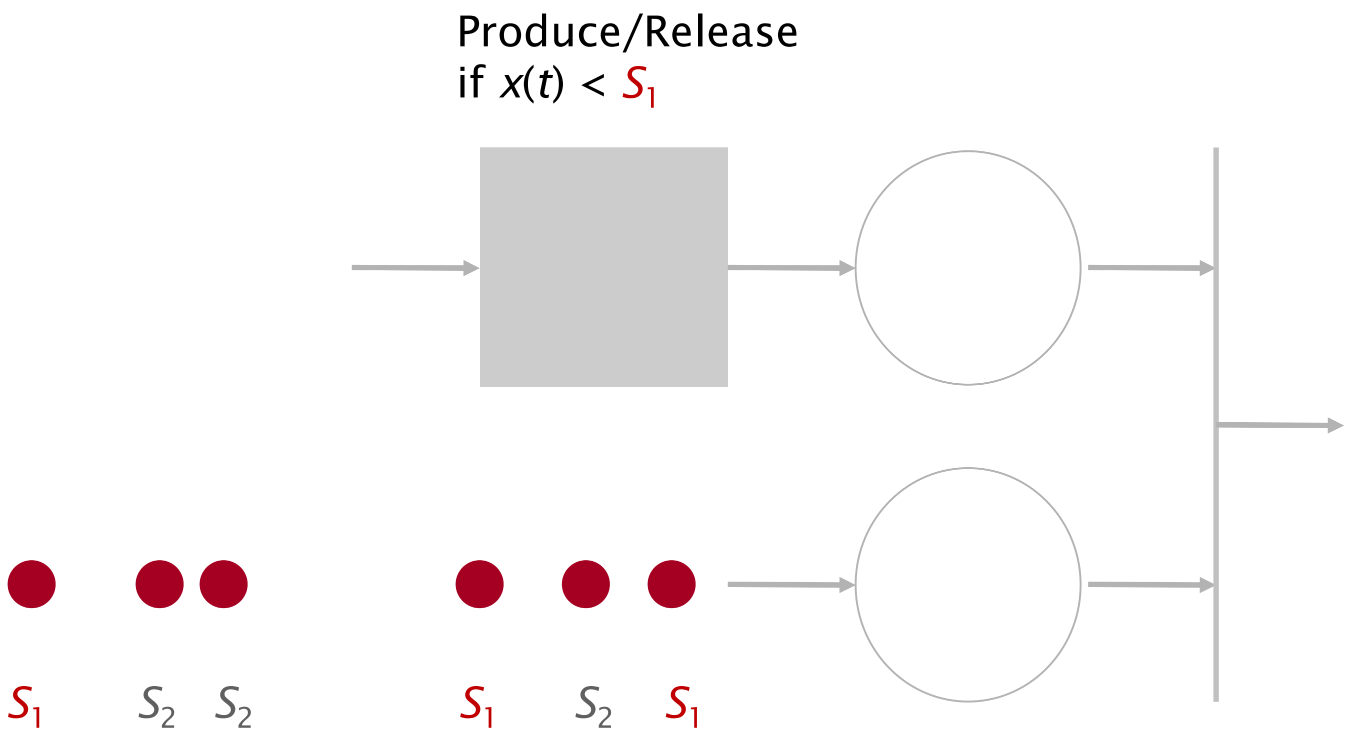


- Use available **data**
- Find a **detection function** that predicts the current state
- Find the state-dependent **base-stock levels** with MP
- Control the system with the state-dependent base-stock policy

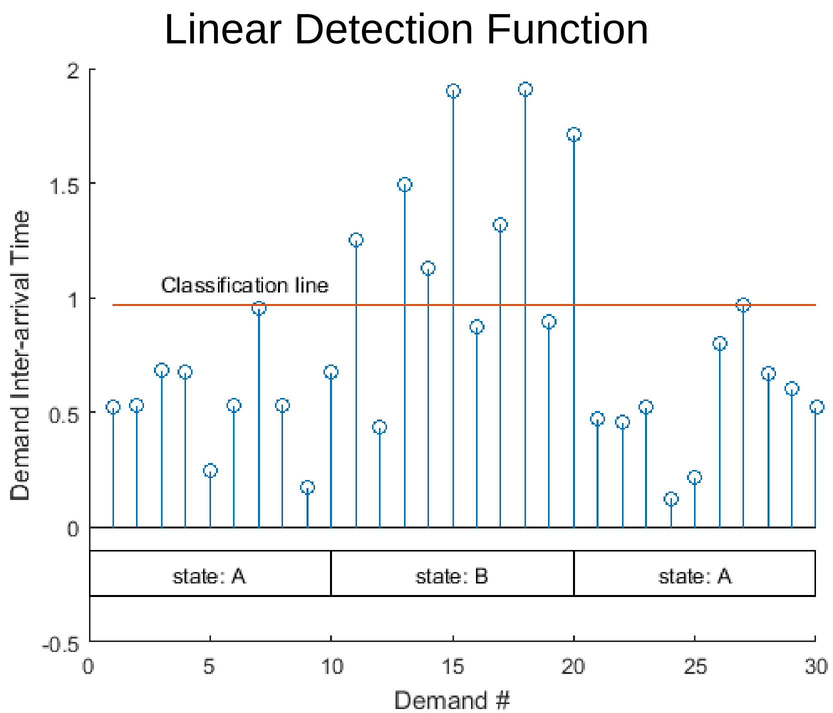
Compare with the **optimal** performance

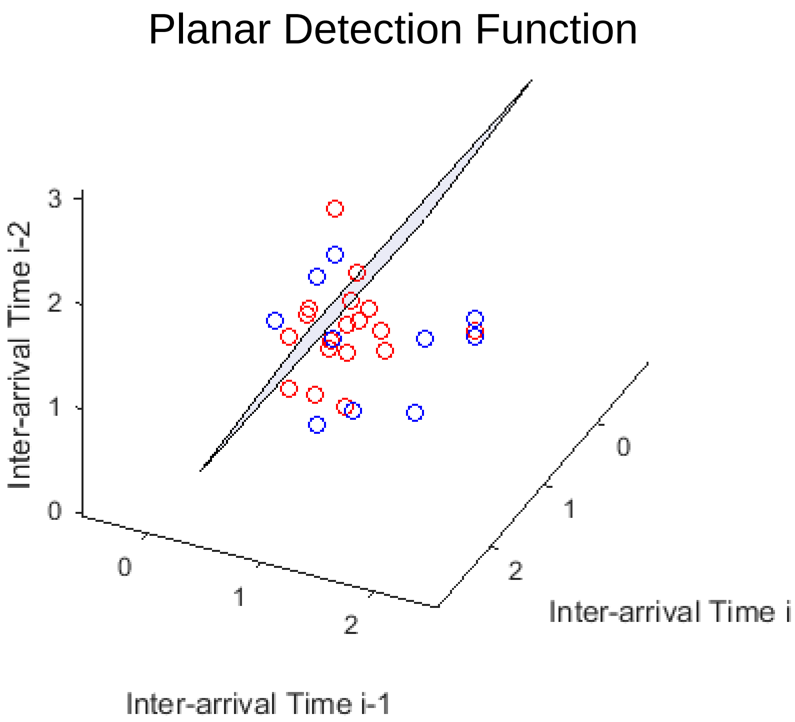
Compare with the single base-stock policy

State-dependent Base Stock System with Observed Data



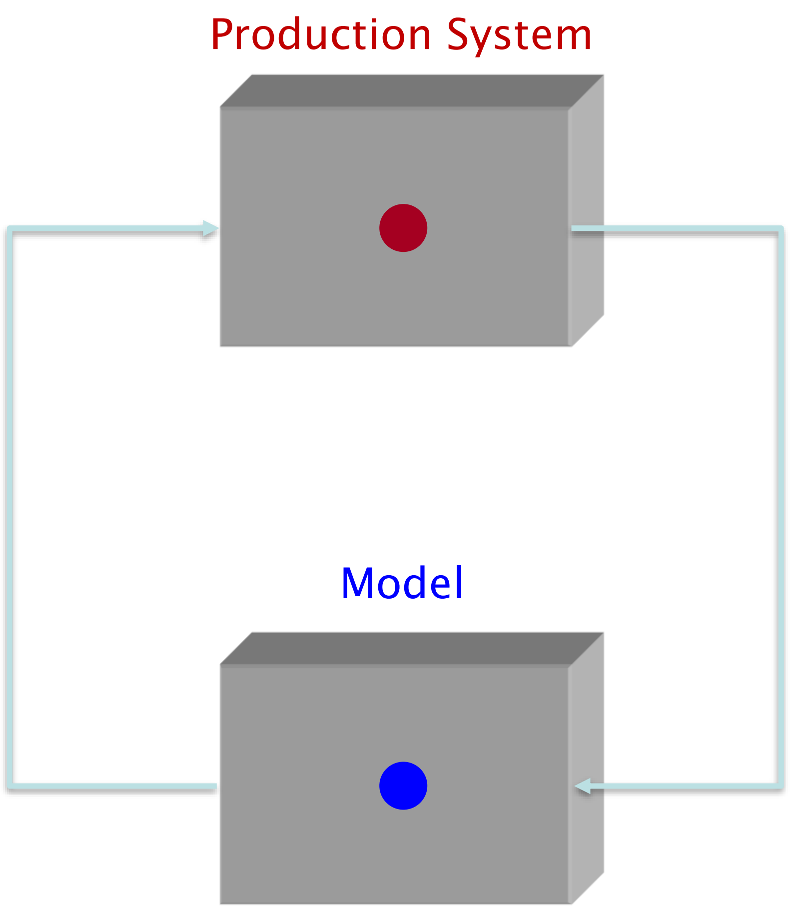
State-dependent Base-Stock System with Observed Data





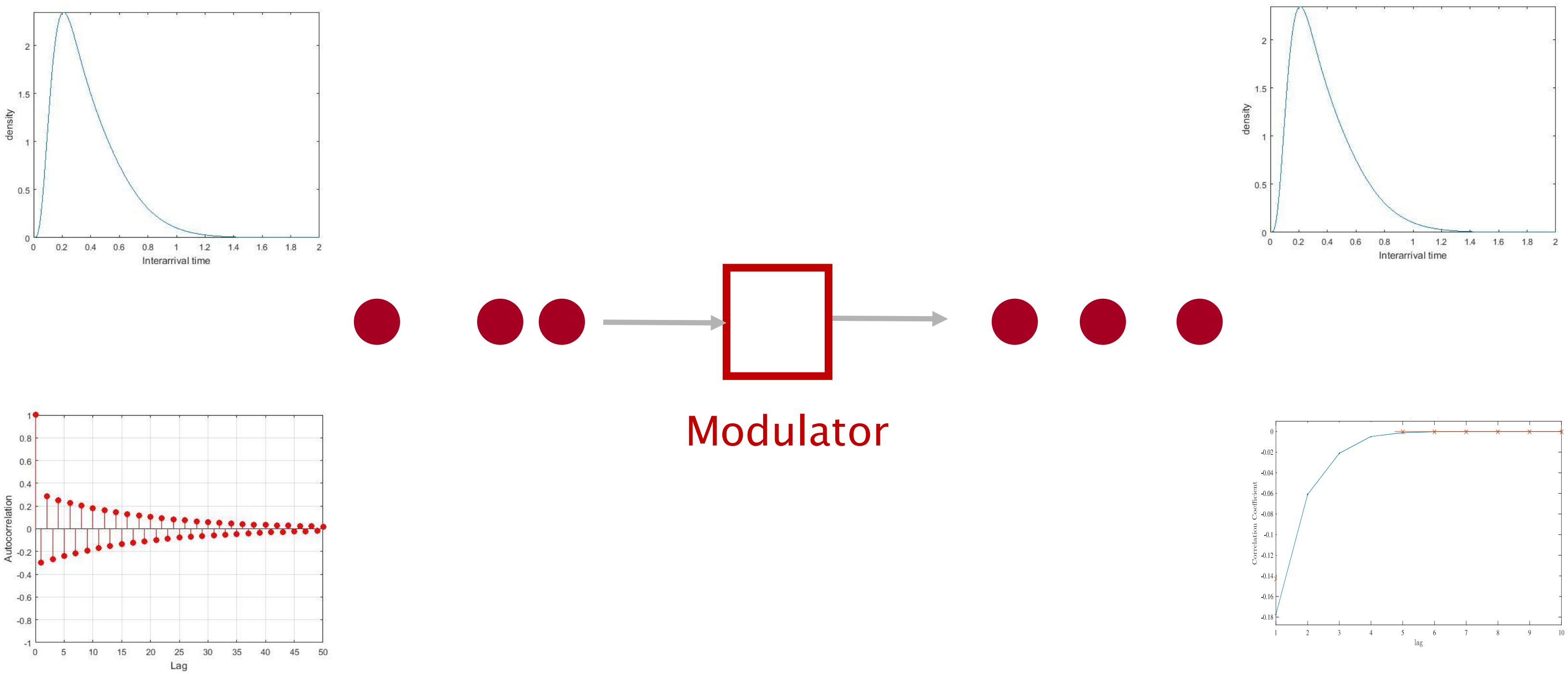
Base-Stock Policy	Detection Function	Total Cost	Improvement
Single	-	126	
State-Dependent	Linear	100	21%
State-Dependent	Planar	88	31%

Research Questions
Data-Driven Modelling and Control



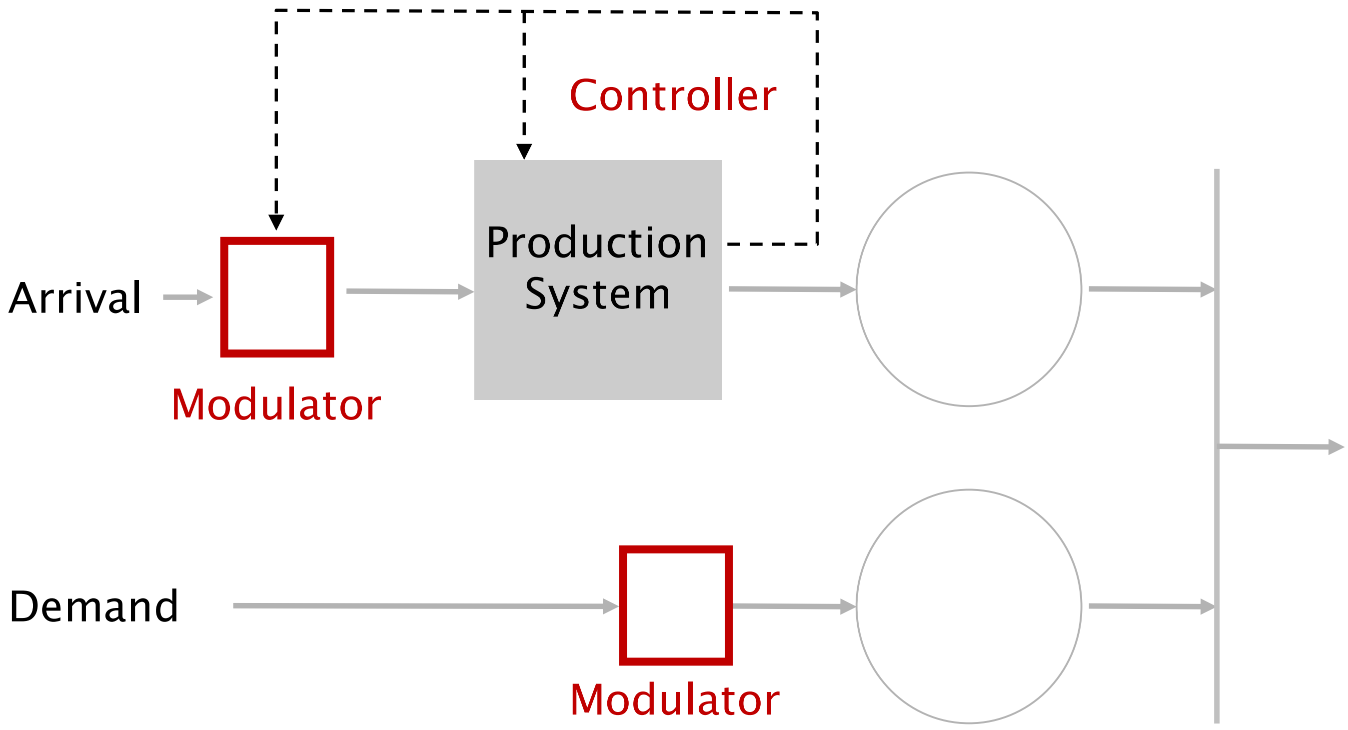
- **Fitting** models based on observed **data**
- **Controlling** production systems effectively by using models that capture output dynamics
- Controlling production systems by **modulating** the input arrival streams

Modulation

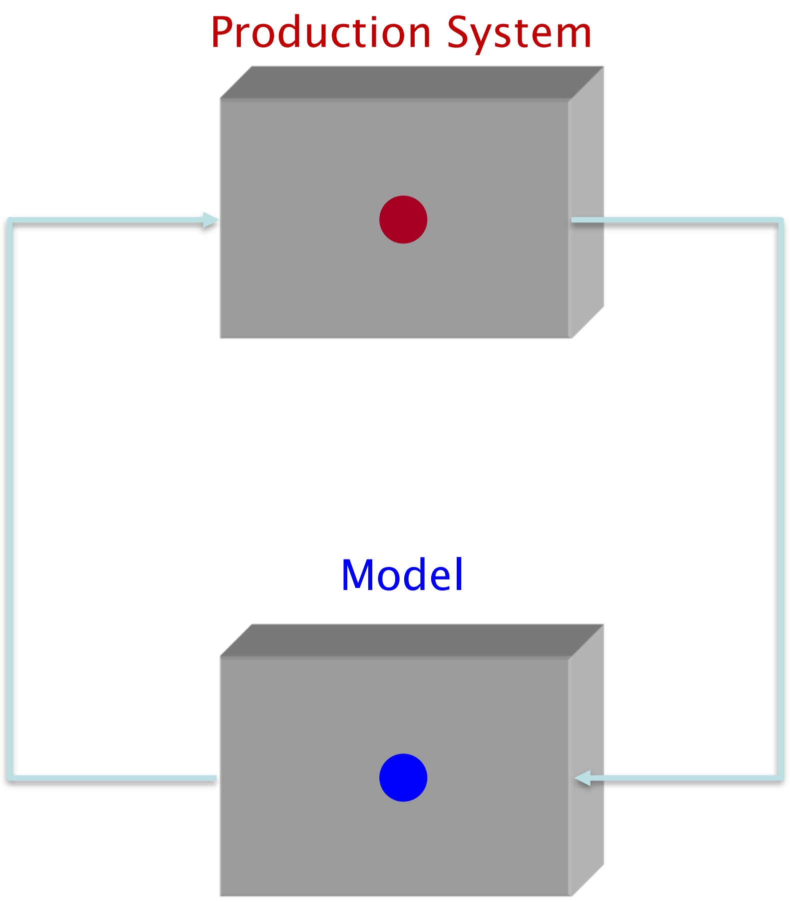


Is it possible to improve the **performance** of a production system by modulating the arrival stream?

Data-Driven Modelling and Control

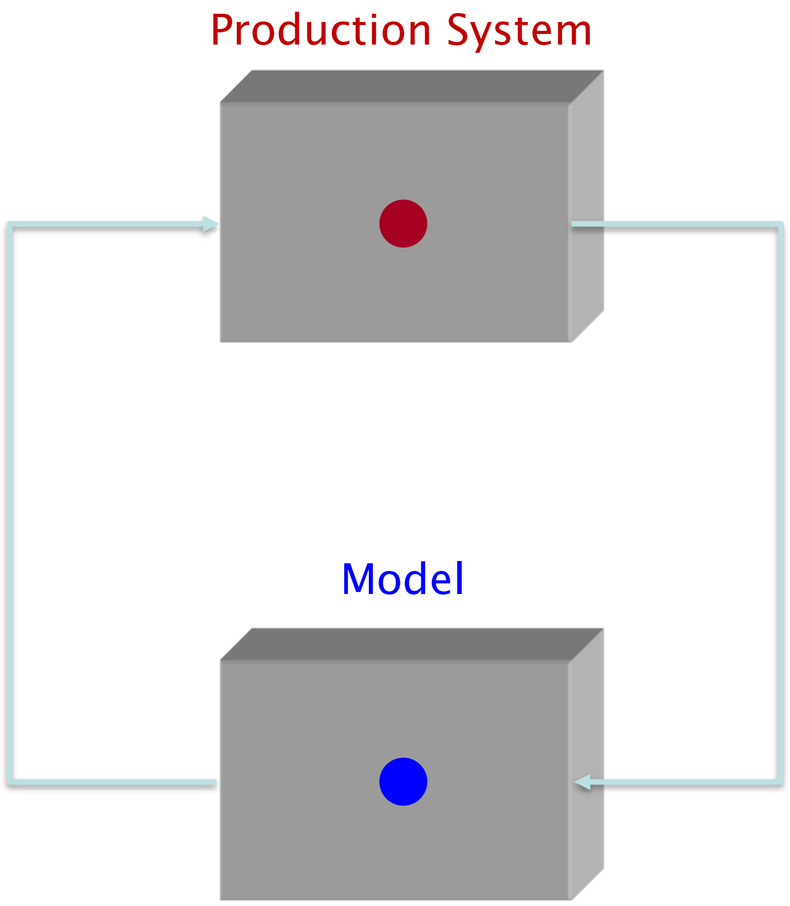


Conclusions



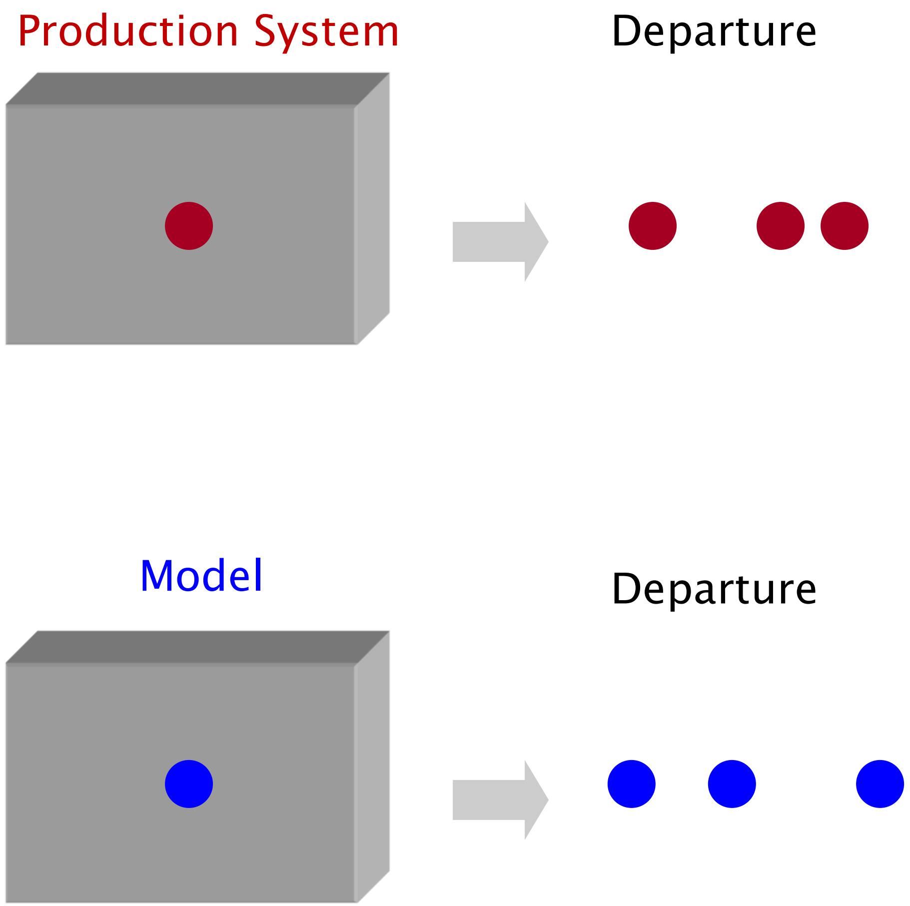
- **Output dynamics** from production systems can be captured effectively by using MAPs
- **Analytical methods** have been developed to analyze the statistical properties of output dynamics from production system
- **System parameters** affect output dynamics
- **Ignoring** autocorrelation can yield **errors** in performance evaluation and suboptimal results for control problems

Conclusions



- Models can be built by using observed **data**
- Production systems can be **controlled** effectively by using models that capture **output dynamics**
- Production systems can be controlled by **modulating** the input arrival streams

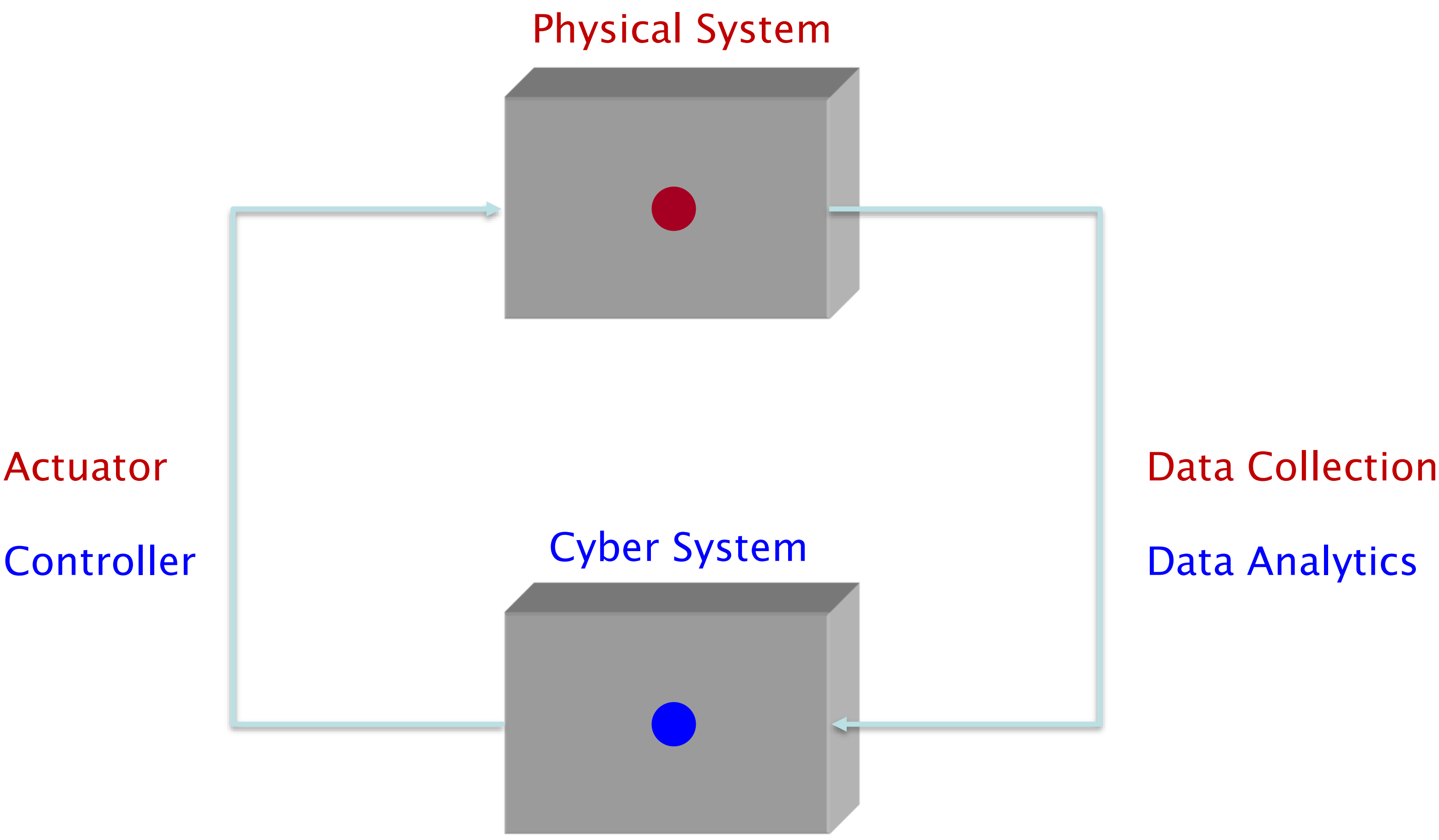
Production System vs Model
"Mathematical Twin"



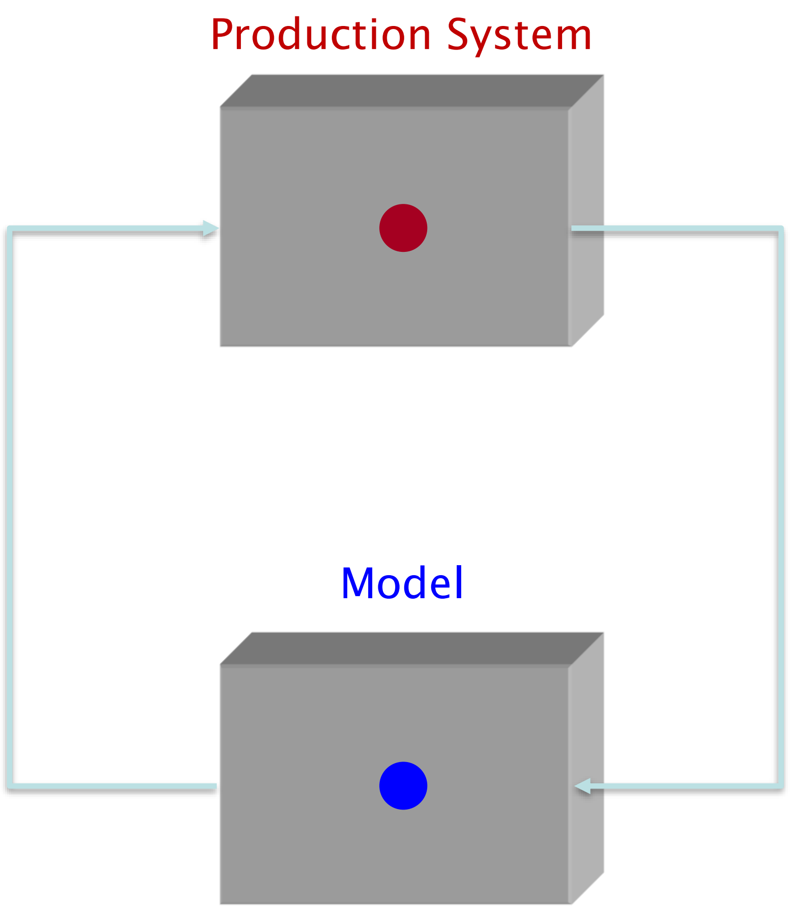
Data-Driven Modelling and Control

"Digital Twin"

Cyber-Physical System

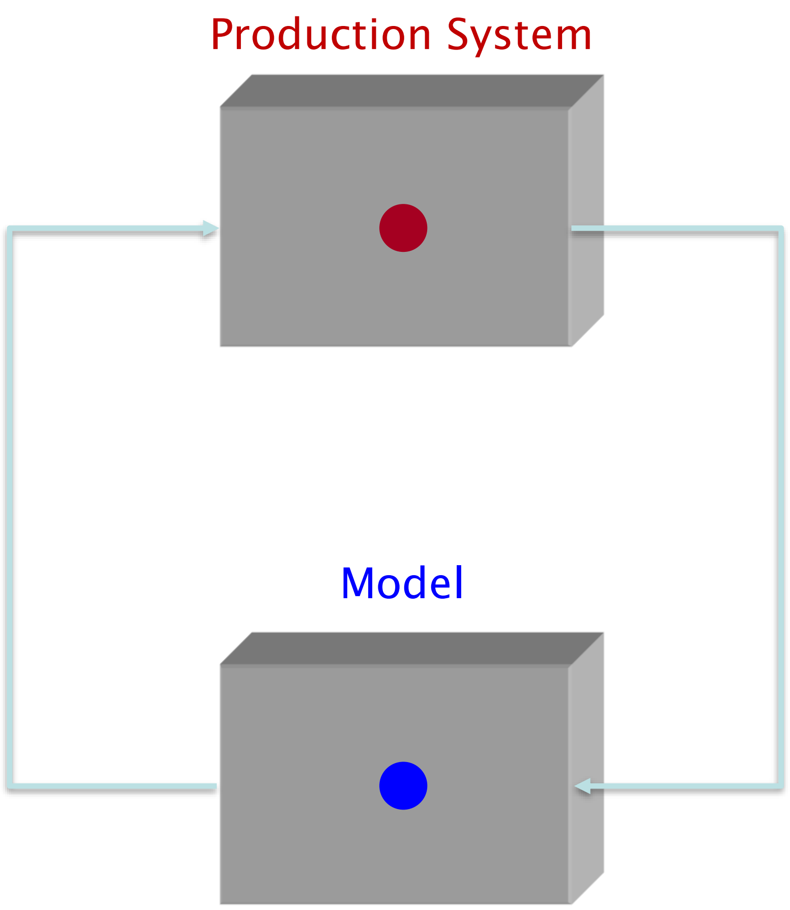


Future Research



- **Analytical methods** to analyze the statistical properties of output dynamics
  - Larger Systems
  - Approximations
- Building models by using observed **data**
  - Methods for Systems with Negative Autocorrelation
  - Joint Simulation-Optimization based approaches

Future Research



- **Data-driven control** of production systems
    - Online state detection and optimization
- **Modulating** input arrival streams to control systems

Modeling, Analysis and Control of **Output Dynamics** of Production Systems

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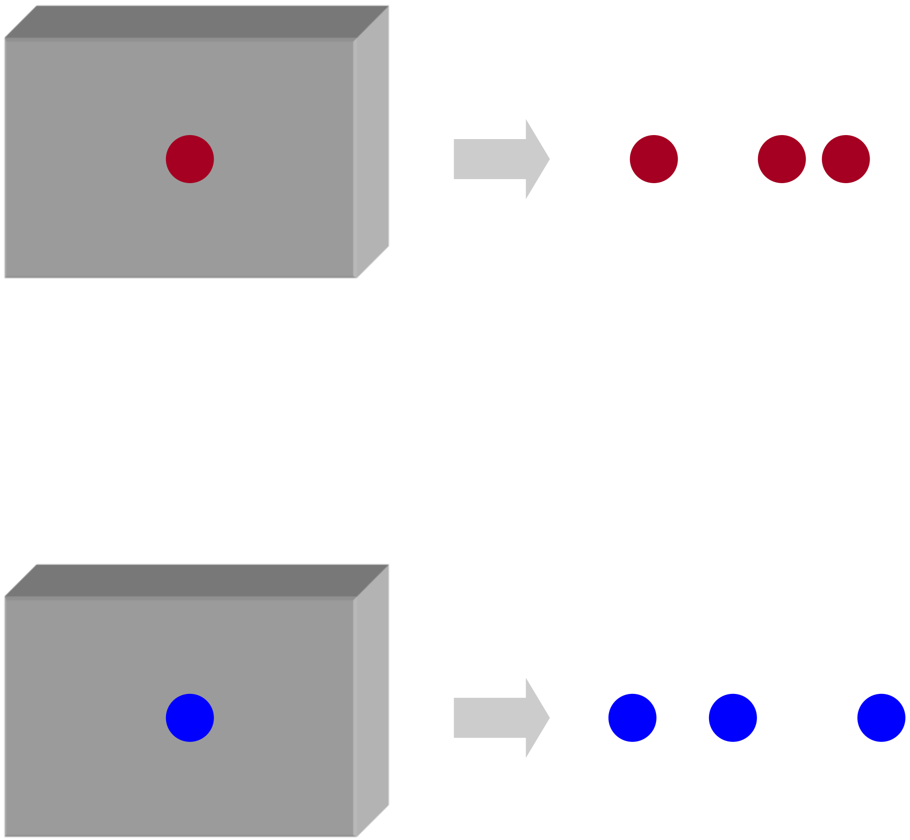

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